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RELATIVITY

AN EXPOSITION WITHOUT MATHEMATICS

By JAMES RICE, M.A.
Associate-Professor of Physics in the University of Liverpool.

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RELATIVITY

INTRODUCTION

For a complete comprehension of Einstein's theory, a special training in the mathematical and physical sciences is indispensable. Nevertheless, it is not impossible, though doubtless troublesome, for anyone to climb the steep slope far enough to glimpse the widening horizon, to guess in an intelligent fashion at what " new thing " the German physicist has offered to us for the interpretation of our world. But we cannot recognise what is new if we do not understand wherein its novelty lies—that is, if we have no proper appreciation of the system of thought on which the fresh principle is imposed—and so we shall have to devote many pages to an account of the origin and development of those ideas about the universe which the principle of Relativity challenges. It is this challenge which the man-in-the-street has in mind when he refers to the principle as "paradoxical," "denying common sense"; and this assertion is quite correct. Relativity does deny *common sense;* but it does not deny either careful observation of natural occurrences, or the result of mathematical analysis based on the simplest postulates consistent with such observations. The settlement of scientific theory accomplished in the seventeenth century by Galileo and Newton was a " triumph of organised common sense. . . . It grounded itself upon what every plain man could see with his own eyes, or with a microscope of moderate power. It measured the obvious things to be measured, and it generalised the obvious things to be generalised."* Thus arose " scientific materialism," and this inheritance

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* A. N. Whitehead, *Science and the Modern World* (Camb. Univ. Press), p. 161.

of "organised common sense" (a phrase of Huxley's) has been regarded (and rightly so) as such an outstanding achievement of the human mind that it is little wonder that the average man is instinctively hostile to anything which challenges its authority.

Howbeit, the past fifty years have seen the development of so many complexities regarding space, time, matter, and energy, that it is clear to the trained man of science, at all events, that the "simple security of the orthodox assumptions has vanished."* In that time the skill of the experimenter and the ingenuity in design of physical instruments have so far advanced that we now possess a considerable knowledge of the facts of Nature in regions far removed from the ordinary experience of mankind. Now common sense is founded on common experience; so to say that something is opposed to common sense is not necessarily to say that it is opposed to truth. It may be quite true to facts which lie outside the possibility of direct verification by the layman, but which have, nevertheless, been observed and subjected to critical research by men of science trained in a special technique and equipped with instruments concerning whose powers of profound and precise measurement it is almost impossible to convey any satisfactory information to those who have not acquired some knowledge of physical science.

Apart from this, the reader should remember that his conceptions concerning space, time, and material (which he regards as so sacrosanct that anyone questioning their validity is regarded as questioning the obvious) are themselves barely three centuries old. Just three hundred years ago Galileo and Kepler were "defying reason"; and the world had to wait another half-century for the genius of Newton so to correlate the work of his predecessors and to enrich it with his own observation and imagination that the minds of men could at least break away from the last traces of medievalism in science and accept as

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* A. N. Whitehead, *Science and the Modern World*, p. 161.

"reasonable" a revolution in ideas about the universe far more catastrophic than that change in outlook to which men are being urged at present.

The author of this little book has no illusions about "making Relativity easy to understand." He has vivid memories of his own early efforts to become dexterous with unfamiliar concepts; he has also watched the struggles of students in his own classes to accustom themselves to the new point of view. But although there is no "royal road to Relativity," the road is not impassable for anyone who is prepared to devote time and thought to the subject. As already stated, lack of mathematical knowledge is not a bar to some comprehension of the principle involved. It may be as well to correct here a popular misapprehension concerning the mathematics of Relativity. It is generally believed to be "very hard." Now, it is no harder in comparison with elementary algebra and geometry than is a treatise on machinery when compared to a popular book for boys on locomotives. It is entirely a question of maturity of mind and experience. The average man nowadays has some acquaintance with the elements of algebra and geometry. Quite a considerable number of young people in the Universities, and even in the higher forms of our secondary schools, are mastering the elements of the calculus, and finding it as easy for their minds at eighteen years to grasp as was the solution of algebraic equations at fourteen. Now, the step from the ordinary calculus to the *tensor calculus* (as the mathematical method suitable for the complete development of the Einstein theory is called) is no greater nor more difficult for the more mature mind to make than is the step from simple algebraic processes to the processes of differentiation and integration for the lad of eighteen. To return, however, after this digression, the reader (with or without mathematics) must be prepared to consider carefully and critically unconscious prepossessions in favour of certain beliefs, and to accept statements as to the evidence which physical experiments of a searching and precise nature bring to bear on those beliefs. In short, a good deal must be done in clearing the ground before we can begin to build the edifice. It is for this reason that the majority of readers have found magazine and newspaper articles on Relativity so unsatisfying. Excellent as some of these have been, it is, in the nature of things, impossible in such a brief space even to begin to show why the older views were at fault, let alone suggest new ones to take their place.

Peculiarly enough, it was the discovery that light in its transmission did not satisfy the requirements of a limited kind of relativity, known as "mechanical relativity," which initiated that series of experiments upon whose well-attested results Einstein founded his postulates. And yet in a sense this was not so peculiar; for, as a matter of fact, this mechanical relativity was already inherent in the Newtonian scheme, and what Einstein's imaginative genius grasped was that this kind of relativity was too limited in its scope. His proposal was an extension to a much wider domain of a principle already present in embryo, rather than the introduction of an entirely foreign and hostile element. This should serve to forewarn the reader against the belief, fostered in quarters where sensationalism pays, that Einstein's work in some mysterious way has destroyed Newton's. The absurdity of such a suggestion will only be too apparent as we proceed. Two centuries of experiment and mathematical analysis lie between the two men, and Einstein stands on the shoulders of the greatest scientific man who has ever lived.

It will give the reader some inkling of what is to follow if at this point we explain the previous paragraph more fully. First of all, what is the "mechanical relativity" there mentioned. Well, it is so simple (apparently) that it is one of those statements accepted as axiomatic, and to question it appears rank nonsense. Suppose a car is travelling along the road at ten miles per hour. A second car overtakes it at fifty miles per hour; then this second car passes the first at a relative speed of forty miles per hour. That is, as stated, accepted as "common sense." At all events, it is an illustration of the kind of relativity referred to. Now let the reader ask himself this question: What is meant by that forty miles per hour. He knows what is meant by the ten or fifty. The lengths *measured along the road* travelled by each car in a given time, when divided by that time, yield those results. There is a definite measurement (*i.e.*, experiment) and a definite mathematical process. How has he arrived at the forty? By a mathematical process—subtraction. But where is the experiment? Now, dear reader, do not at this point get irritable and begin talking about hair-splitting and quibbling. It is just these cherished notions of yours which must be subjected to a close scrutiny. Ask yourself fairly and squarely: Where is the experiment? Do you know of anyone who ever attempted to measure the rate at which the distance of a car ahead of his car was increasing? Have you any idea of the manner in which such an investigation—or any similar one—could be carried out, or of the degree of precision which could be attained in it? Do not imagine that the truth of the statement is being questioned; you are merely invited to consider if you know the evidence for it, remembering that the statement means that if the person in the slow car could himself measure the distance of the first car ahead of him *without recourse to the road measurements*, he would find that it was receding from him at forty miles per hour. The italicised words are important. The fast car might carry one end of a long tape measure, which was thereby pulled (in a tight condition) over a mark on the slow car; that would constitute a *direct* observation by the observer in the slow car on the fast. It would not depend in any way on any measurements made on the road, let us say, by the police in the interests of public safety! If you have realised that a certain mathematical step has been taken in obtaining that forty, and that you are a bit doubtful about the evidence for it, you have made one important step in your journey. There is, indeed, good evidence for it, but it is generally buried, as far as the ordinary man is concerned, in works on physics, especially on dynamics, theoretical and experimental. The evidence is entirely indirect, and consists in the fact that the assumption that this mechanical relativity is true forms an organic part of those dynamical principles first clearly formulated by Newton, which have been so signally successful in embracing the behaviour of moving bodies on the surface of the earth, and of the planets in the heavens.

Now as to the question of the behaviour of light and its bearing on this mechanical relativity. Since the time of Newton, until quite recent years, the transmission of light has been regarded as a purely mechanical process. Newton regarded it as a rectilinear flight of minute corpuscles ejected in a continuous stream from luminous bodies. The nineteenth century abandoned this view, but still retained a mechanical explanation. To the great physicists of that century, light-transmission resembled an undulation in a vibratory medium, and so they postulated such a medium—the ether—as filling all interstellar space, and even all space between the ultimate particles of matter. Furthermore, experiments on the speed of light transmission became so precise that an experimental genius like the famous American physicist, Michelson, could obtain the time required for a flash of light to traverse a length of a few hundred feet with an accuracy of one part in 10,000. This amazing feat enables us to state that light travels 300,000 kilometres per second. Such a speed is on quite a different scale of magnitude to the movements of terrestrial bodies, or even the speeds of the planets in our solar system. (The earth's speed round the sun is about thirty kilometres per second.) Nevertheless, apart from this great difference in magnitude, the transmission of light was regarded as of the same nature as the travelling of a body or an undulation, and was tacitly assumed to conform to the mechanical relativity referred to. Now consider what this implies. Let us liken the earth to the slow car, and a beam of light to the fast. If the light is overtaking the earth, as it were, it should travel past it just a trifle more slowly than if it were meeting it directly. Michelson's apparatus was just precise enough to measure that trifle. There was no possible doubt about that. But the trifle obstinately refused to manifest itself. Apparently the light passed the earth *at the same speed*, no matter what direction it took. Naturally the leading theoretical physicists of the time applied themselves with assiduity and skill to the explanation of this awkward trifle, whose strange disappearance was more than a trifle awkward! There were a number of ways of evading the difficulty. The most obvious way was to deny the power of the apparatus to make such a refined measurement with sufficient precision. But there is no escape that way. No experimental physicist of any standing in his craft has ever ventured such a suggestion. In fact, it is notorious that repetitions of the experiment by Michelson himself and others have only served to justify the claim that at the present day, some forty-five years after the first attempt, the apparatus is easily capable of measuring an effect even as small as one-tenth of what might be reasonably expected as the result of the assumption that mechanical relativity is true for light-transmission. Another explanation of the obvious type would be to admit that the experiment proves that the earth is absolutely at rest in the ether—that is, in space. But such a return to pre-Copernican geocentric views would be simply unthinkable. It would involve our whole science and cosmology in such a catastrophe that, compared to it, the modifications required by the Einstein theory would be as the tremor in a house, caused by a passing vehicle, to an earthquake.

To be sure, no one has ever ventured to offer either of these suggestions. For two decades after Michelson's first experiment two explanations were earnestly discussed by the leading minds in physical science. The first reminded us that in some ways the ether resembled a fluid so perfect that, although a planet might travel through it without appreciable resistance, it might carry along with it some of the ether as a surface layer—a kind of ethereal skin—thin, no doubt, but still thick enough to make it true to say that the earth was at rest *relatively to the ether in its immediate neighbourhood.* Speculations of this type had, indeed, been current in the literature of physics for some years before Michelson's experiment, and, as a matter of fact, Michelson's own belief in those days was that he had verified them. Nevertheless, it was found to be exceedingly troublesome to prove by dynamical methods that such an ethereal layer could really cling to the earth in the manner suggested, neither tearing away, as it were, from the terrestrial surface, nor, what was even more important, slipping past it. It could only be done by attributing properties to the ether for which there was no justification from any other physical experiment. Now, there is nothing more suspect by the genuine man of science than a hypothesis which won't work unless it is complicated by an addition of minor (so-called *ad hoc*) assumptions, which find no other support than that they are required to make this hypothesis work. He may have to be content with it for a time, in default of something better; but sooner or later he will either find the necessary support, or abandon the idea entirely in favour of something better. In the case in question, something better did turn up very quickly. An Irish physicist, Fitzgerald, suggested that, although the earth might travel through the ether without disturbing it, as it could easily do on account of the molecular and atomic structure of matter, yet it was distorted by the motion from the shape which it would have were it at rest in the ether; that, in fact, the length of everything on the earth, no matter what its chemical constitution might be, is shortened, in a direction parallel to the earth's true motion in space, by an amount which depends entirely on the ratio of the speed of the earth to the speed of light, while the dimensions of any body at right angles to the motion remain unchanged. Thus a sphere would be flattened into an oblate ellipsoid with its short axis parallel to the earth's velocity, and the flattening would be greater the greater the velocity. Fitzgerald was able to show that if one accepted this view and prescribed a certain formula for the amount of shortening, the changes in size in Michelson's apparatus, as he rotated it into various directions in his measurements on light-beams travelling in these directions, were just of such an amount as to explain why the measurements failed to yield the earth's drift through the ether and why it was hopeless to look for any positive result in that way. This suggestion of Fitzgerald's proved extremely helpful and fertile in promoting fresh theoretical and experimental work, although the complete working out of all its implications was, in the main, due to Lorentz, a Professor of Physics at Leiden, and Larmor, an English mathematician at Cambridge. This hypothesis, like its predecessor, was of the *ad hoc* variety; but various experimental physicists set to work to remove this stigma from it. Such a change in dimensions when a rod is turned in different directions, as Fitzgerald had suggested, was no doubt extremely small in value; nevertheless, no one felt that it was satisfactory to evade the difficulty by the plea that physical instruments, at the standard of precision attained even then, could not detect it. Of course, there could be no question of trying to measure directly the length of a rod with a graduated rule, no matter how precise; for the rule would have to turn with the rod from one direction to another, and would just suffer the same change in length, so that agreement of certain marks on the rod and rule would not be even theoretically disturbed, quite apart from the ability to perceive such a disturbance, did it actually take place.

But there are a number of electrical and optical properties of a body which are altered by a compression or extension of a body, and the instruments for detecting such alterations are quite precise enough to find them for changes quite as small as might be anticipated, if Fitzgerald's hypothesis were correct. So some four or five experiments of this type were performed with great precision by a group of English experimentalists in the first five years of the present century. Alas! not one of the expected effects manifested themselves, and once more no doubt could be thrown on the skill of the investigators nor the adequacy of the instrumental equipment.

This was a disturbing state of affairs. Perhaps the average man may think it astonishing, and even ludicrous, that anybody should be concerned because he failed to recognise that a particular rod or wire had altered by about a one-hundred millionth part of its own length when turned from one direction to another. "Surely," he will say, "this is precision of measurement gone mad; what on earth can it matter in any practical way if one fails to observe such an exiguous quantity?" Such a criticism overlooks that rigorous scientific honesty which characterises the real man of science. Newton for many years refused to publish his famous law of gravitation because he believed that its results were opposed to the facts concerning the moon as he understood them to be. Measurement had not reached anything like the precision to which it has now attained; nevertheless, the facts were apparently against him, and not until he was satisfied by later and more accurate investigation that his work was supported by them would he consent to put forth any theory. So, more than two centuries later, the facts, however minute, were apparently against all the explanations advanced in connection with the failure of light to comply with the principle of mechanical relativity; and it would have been no more disturbing if it had been a question of a hundredth instead of a hundred millionth, for the instruments were quite capable of dealing with a thousand millionth. There could be no sheltering behind inadequate equipment or skill. Somewhere, then, there was a flaw in the reasoning, or a false postulate.

Mankind loves to exalt the possessors of great minds to a lofty pinnacle, so that it can indulge in a modern form of idolatry, forgetting too frequently the absolutely indispensable spadework of those whose achievements do not lend themselves to that love of sensation and novelty which marks our age. Do not let us, however, in our entirely justifiable admiration of Einstein's brilliant genius, overlook the patient labour of those other searchers for the truth, without whom there would have been no Einstein, just as without Kepler and Galileo there would have been no Newton. Of this little band no name is more honoured than that of Lorentz, the Dutch physicist, whose contributions to the solution of this serious difficulty were the foundation on which Einstein began to build. Lorentz actually did solve the problem in a manner that caused no serious breach with the past. In 1904 he published a paper which gave a satisfactory account of the reasons inherent in the fundamental electrical actions between the ultimate parts of matter, which, when combined with the hypothesis that not only matter in bulk, but also these ultimate parts experience the Fitzgerald contraction owing to motion through the ether, successfully conceal all evidence of that motion. It was an exceedingly skilful *tour de force,* and a fitting end to what had been one of the most interesting periods in the history of physical science. But hardly had the work of this veteran of science been assimilated by those interested when there appeared, early in 1905, a paper in the German *Annals of Physics,* written by a young man of twenty-six (who, as became known afterwards, had not even read this latest contribution of Lorentz), which put a completely new complexion on these difficulties and their solution. From the point of view of mathematical symbolism, there appeared to be a formal agreement between his solution and that of Lorentz, but in reality the underlying conceptions were entirely novel and revolutionary, and, what is more, as the young Einstein was able to show in later communications, Lorentz's solution actually carried implicitly in its mathematical formulæ a denial of the relevance of the ether or absolute motion through space in the investigations in question—a point which had escaped even the acute mind of Lorentz.

Einstein had, by one of those flashes of insight which are the prerogative of genius, realised that something deeper than suitable hypotheses about the properties of matter or skilful mathematical analysis was involved. He saw that our traditional attitude towards the concepts of space and time required changing once more, just as they had been changed two or three centuries earlier in those days when Galileo and Newton had begun the work of the first scientific synthesis.

This brief introduction must serve to give the reader some inkling of these immediate and direct causes which gave rise to the promulgation of the Relativity principle; but he will realise, from what has just been said, that no attempt to understand the principle will be successful unless he is prepared to examine carefully his notions of space and time—not only how they have arisen, but also what preceded them in human history. The average man thinks that he knows all that is necessary about these troublesome things. (He ought to tell a philosopher that and then " take what came to him.") He probably also thinks, if he can repeat the well-worn three phrases which embody Newton's laws of motion, and knows enough mathematics to understand Newton's formulæ for the law of gravitation, that he is sufficiently well equipped in these matters for any practical purpose. Well, it is more than doubtful; but, at all events, I hope that my readers will have the patience to follow me through three or four chapters of indispensable initiation into these important questions before making the attempt to grasp the essence of Einstein's work.

CHAPTER I

SPACE

In a sense our problem goes back to the beginnings of astronomy. To the ancients the idea that the earth was a massive fixed thing, round which circled daily the celestial sphere with its multitude of light bits of flame, was so obvious to the senses that a few hardy spirits who suggested that the sun might be the centre of all things were derided as denying plain facts. Even the boldest thinkers had not conceived that these stars were massive bodies of similar constitution to the earth: the telescope had not yet arrived.

The greatest speculators on these matters were the Greeks. To them the *uniform, circular* movement of nearly all the celestial bodies was evidence of that underlying harmony, order, and beauty which they saw in all Nature. While coarse, heavy, terrestrial matter could move about in all sorts of unregulated and violent ways, only the perfectly even movement in the perfect curve—the circle—could be possible to those bits of starry flame, which were the most striking and direct examples of natural perfection. But there were seven of these bodies which marred the extreme simplicity of this perfect picture. They were the sun and moon, and the planets (" wanderers ") Mercury, Venus, Mars, Jupiter, Saturn. Let anyone, free from all sophistication about the diurnal rotation of the earth, observe the daily motion of the sun. It goes round the earth in a circle in one day—nearly. It really goes round the earth in a spiral, and it takes on an average about four minutes longer to travel one turn of the spiral than it takes any one of the stars to travel its circle: in six months it spirals from south latitudes to north, and returns back along its spiral in six months. The moon behaves even more irregularly, while the five planets, also executing rather irregular spirals over many months, are, during certain periods, travelling a turn of their spiral faster than the stars—" direct motion "—and during others more slowly, just like the sun—" retrograde motion." Still, the Greek astronomers managed to save the situation. Circles whose centres were not situated on the earth were used, and when that failed the ingenious " epicycle " theory was invented. The planet travelled uniformly in a circle, whose centre also travelled uniformly on the circumference of a second circle, whose centre might be fixed, or, if necessary, in its turn might sweep around in a third " perfect path "; and so on. The reader must have no misconception about this process. The movements of sun, moon, and planets could be *described* in this way with all the accuracy required for the degree of precision in observation then possible by using a sufficient number of epicycles. This method was highly developed by the great Alexandrian astronomer and geographer, Ptolemy, and his medieval successors. Actually there is nothing theoretically wrong about such a procedure; it could be applied now, only we should require many more epicycles, and thus render it extremely clumsy and laborious. There was no question of "forces" acting between sun and planets in those days. In so far as forces were thought of at all, there were vague, animistic views current of supernatural powers moving the celestial sphere in its daily round. The Greeks were great geometers, and to be able to give an exact *geometrical* account of a planet's path, especially in terms of circles, satisfied all their requirements as regards the "explanation" of these phenomena. There was, naturally, no conception of an infinite space and of endless multitudes of stars. Distances such as we conceive now would have been laughed out of court as incredible. Nevertheless, a few unorthodox men did point out that if the sun were taken as the "fixed" reference point, the description was much simplified, as fewer epicycles would be required for it. Now, actually, when Copernicus began revolutionising men's ideas in the fifteenth century, it was entirely on these grounds of simplicity. His view was not really novel. Centuries of neglect of the ancient sources of learning and conservatism in the Aristotelian tradition had buried the scattered records of early scepticism. It may seem strange to the reader that the Greeks, in their desire for simplicity and beauty of description, should have refused to adopt such an obvious simplification. Space does not permit us to enter on this interesting topic; suffice it to say that they had a crude sort of physical science, which had to be satisfied as well. Thus, if the sun were really fixed and the earth went round it and also turned on its axis, a body thrown upwards would come down to the west of its point of departure, and all things like the air, flame, and other light objects would part from the earth, left behind in the rush through space.*

Now, reasons like these were cogent enough. Very many people nowadays would find it hard enough to give a really satisfactory explanation of the absence of any *obvious* westerly deviation of a projectile; it really involves a genuine understanding of dynamics. Frankly, Copernicus quietly ignored these difficulties, and put forward his new (and yet old) hypothesis solely as a gain in *mathematical simplicity*—instead of eighty epicycles, only thirty-four were required; but the world had to wait for a century before Galileo, with his telescope and his laws of falling bodies, and Newton, with his dynamics and gravitation, supplied a new physics for the new astronomy. All that Copernicus had to offer in addition to this simplicity was a kind of mystic appeal to our sense of the gloriousness of the sun as a celestial object. Yet his idea gained ground. Why should it have lain unnoticed for more than fifteen centuries, to come forward with comparative rapidity now? Well, it was an age of restless activity in all directions—in the world of politics, religion, and travel. The book just referred to will give the reader a vivid picture of the mental unrest of the time. Naturally, it did not end there. Copernicus was so much fettered to the past that he could never free himself from description in terms of circles. Not so Kepler, the famous mathematician, contemporary with Galileo. Kepler was equally enamoured of mathematical explanations, and eagerly adopted the heliocentric ("sun-centred") as opposed to the "earth-centred" or geocentric view, but strove to describe a planet's motion in terms of *one* easily defined geometric curve instead of many circles, with, of course, the Sun as reference-point or "origin." He had the very careful observations of Tycho Brahé on Mars as his material, and after many

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* The interested reader should consult E. A. Burtt's *Metaphysical Foundations of Modern Science* (Kegan Paul).

attempts and failures hit upon the ellipse.* (Peculiarly enough, Kepler never considered this epoch-making discovery of his as important as other, now forgotten, geometrical discoveries concerning the dimensions of the solar system.) This gave the deathblow to geocentricism. The sun was the fixed centre of all things. The telescope of Galileo had shown the valleys and mountains of the moon, the phases of Venus. The planets were worlds like ours. The stars fixed in the firmament appeared to move because the earth rotated. Nature exhibited a beautiful and simple mathematical order, one of whose most prominent manifestations was summarised in Kepler's famous three laws of planetary motion. Firstly, every planet moves in a plane curve, the ellipse, the sun being at one focus. Kepler had foreseen that in forsaking the circle, all question of uniform *angular* motion would have to be abandoned. Yet he had to save uniformity of motion somehow. To his delight he found that, although the line from sun to planet did not sweep out equal angles in equal times, it did sweep out larger angles as the distance decreased in such a manner that it swept out equal areas. This is his " law of description of equal areas." It is well known that the outermost planets require longer times to travel once round their orbits. Kepler discovered that the squares of their periodic times varied in proportion to the cubes of the longest diameters of their orbits. (Third law.)

Once more the reader is asked to observe the essentially *geometric* nature of all this explanation. Dynamical astronomy was still to come. Galileo had laid the foundation of dynamical science. It remained for Newton to develop it and achieve his scientific supremacy

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* If the reader fastens the ends of a loose piece of thread to two drawing-pins stuck in a board, and keeping the thread tight with a pencil, moves the latter round, it will draw an ellipse. The two pins are at its " foci." In short, the sum of the distances of the two foci from any point on an ellipse is always the same.

by his applications of it to planetary and terrestrial movements. Men's ideas had, indeed, widened, but as regards their notions of *space,* they had merely passed from a sense of a limited region surrounding a *fixed* earth to that of a limited region surrounding a *fixed sun.* Newton swept away both and gave us infinite, absolute space in which *all things moved,* and *absolute time* which "flowed" independent of all natural phenomena. When the reader thinks of these conceptions as "plain," "common-sense," let him remember they are not three centuries old.

CHAPTER II

THE NEWTONIAN THEORY

Newton's work was based on his introduction of the concepts of *inertia* and *gravitation,* and the close connection which he discovered between these two properties of matter. It is true that he was, as regards the former, somewhat anticipated by Galileo, who clearly recognised that force was required to *change* motion, not merely to *maintain it,* a view which seemed to contradict flatly the most obvious feature of motion on the surface of the earth. However, Newton's famous three laws of motion defined in a precise and *quantitative* manner what inertia meant, and his equally famous law of gravitation embodied the relation which he found to exist between the two properties. It is very necessary that we should study these matters closely enough to recognise what the term "inertia" really means, and what is the nature of this relation; for, although Einstein has proposed a different law of gravitation, he has carried over this relation from the older law to the new, and it will be impossible for anyone to have the faintest idea of the latter unless he has grasped the implications of that close connection which Newton postulated in his teachings.

INERTIA.—Let us take a very simple illustration. Everyone is familiar with the fact that it requires a greater effort on the part of man or horse to start the motion of a vehicle on horizontal ground, than to maintain that motion at a steady pace, once started. We recognise that the effort involved in the second case is required to overcome the friction of the ground, and will vary with the nature of the ground's surface. This also operates during the initial period when the vehicle is "getting up speed"; but in addition a pull is required to "overcome the inertia" of the vehicle, as we say, to accelerate the speed from zero to the steady value maintained. A similar additional effort is required during a period when the speed of the body is being increased from one steady value to another. If we cease to pull, the motion does not cease on the instant; it takes time for the opposing friction to "overcome the inertia" once more. If we wish to bring the vehicle to rest more quickly still, we must push backwards to do so; on sufficiently smooth ground we might have to exert a backward force of considerable amount to bring the body to rest quickly or to decrease its speed at a sufficient rate. In short, inertia is the name given to that property of the body in virtue of which, apart entirely from its situation or the nature of the surface upon which it may rest, force is required to *change its state of motion*, whether that change be an increase or decrease or simply a change of direction. (As an illustration of the latter change, no body can move in a curve, even at the same speed, without a force exerted on it whose direction is across the path towards that side to which the curve is concave.) So much for the general, qualitative conception of inertia. If we wish to define it quantitatively, we must experiment carefully. There are two ways open to us. We may, in one case, determine precisely the amount of friction involved, and, subtracting it from the total force exerted, determine just that amount of force required *to produce the change of motion;* or we may eliminate the effect of friction by allowing the body to run on a gentle slope, which has just the right inclination to overcome the friction, so that, once started, the body will run *uniformly* downhill without any further influence acting on it. In scientific laboratories either of these methods can be applied to suitably made bodies on suitable tracks with very precise means for measuring the time of travelling and the velocity of travelling at any place on the track.

The results of such investigation yield a very remarkable result. Suppose, for example, we subject two different bodies, which we shall name A and B for convenience, *to the same pull* (friction being eliminated or compensated), and we find that it requires one second for A to attain some standard speed—say one metre per second—while two seconds are required for B to attain the same speed under the given pull; it will now be found that if we apply a different pull to the two bodies A and B, while undoubtedly different intervals of time are involved in which the standard speed is reached in each case, yet they are still in the ratio 1 : 2. For instance, if the pull were smaller, so that A required three seconds, then B would require six seconds. This simple result is universally true. If, under a given pull, B requires n times as many seconds to attain some standard speed as A requires, then under any other pull B will require n times the interval which A requires. If the pull is increased, both intervals of attainment are decreased; if the pull is decreased, the intervals are increased, but they will for a given pair of bodies *maintain the same ratio to one another*. This simple result embodies the essence of Newton's second law of motion, and it was by this ratio that he gave a quantitative definition to inertia, saying that the "*mass*" of B is n times the mass of A. The masses of bodies are, in short, proportional to those intervals of time in which the bodies would, *in unresisted* motion and subject to any pull (equal for all), severally attain some standard velocity. The longer a body takes under such precisely defined circumstances, the more "massive" is it in proportion; its mass is proportionally greater. The reader is asked, before proceeding, to observe most carefully that the idea of the weight of the bodies is nowhere involved in these experiments. No balance, either spring or beam, is used; no weighings are performed. With the boldness of genius, Newton projected these ideas into the universe. On the earth's surface the rough ground masks the effects of inertia by its frictional forces; even the air offers a resistance to that persistence of motion which Newton intuitively perceived to be a universal property of all matter. Not so "empty space." In that infinite void all things move, and without any hindrance from the void; no force is required to maintain this motion of the rectilinear, uniform type. It should persist by reason of the inertia; so this straight-lined, unvarying motion is called "inertial" motion. Yet the heavenly bodies do not present this simple type of movement to us. Confining ourselves to our nearest neighbours, and admitting the rotation of the earth to account for the diurnal movement of the stars, we see that even if the sun were at rest in the void, or moving through it in a straight line, the planets and our earth are certainly not doing either.

Gravitation.—It was here that Newton introduced his second great idea, and once more supported it with the most rigorous calculations which the subsequent two and a half centuries have verified in all but the most minute details. The apple falls to the ground with accelerated speed; the projected body ascends directly with a decreasing speed or describes a curved path; the moon moves round the earth—because the earth is exerting a force across space on these bodies. The planets sweep round the sun because they experience a force towards the sun which changes their direction of motion.* The planets themselves exert forces on their satellites, which keep them in their orbits. Notice how different all this is from

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* That would be the only change if the orbits were circles. Insomuch as they are ovals, the planets also experience some comparatively small changes in speed of motion as well.

the former vague views about the influences required to carry the planets round in their paths. Even Kepler thought of forces directed *along* the orbits. But Newton thinks of forces *across* the orbits, directed towards the sun. He found complete confirmation of this in the facts which Kepler had discovered concerning these orbits. For example, Kepler found that the line joining sun and planet swept out equal areas in equal times. Newton gave a mathematical demonstration that this was only possible (assuming his own laws of motion to be completely true) if the force exerted on the planet *were directed towards the sun.* Kepler discovered that the best geometric description of the motion of a planet round the sun is to state that the orbit is an ellipse, with the sun at one focus. Newton gave a mathematical proof that, if that were so, then the force between sun and planet must decrease with increase of distance apart, " varying (as the common phrase runs) inversely as the square of this distance."* Further, if this gravitation were universal, the change of motion produced in the moon by the attraction of the earth, and the change of motion produced in the projectile near its surface, should be related in a definite way which could be calculated from these premises. It turned out that they were. Moreover, Newton saw that if his inverse square law were true and all the planets acted on each other as well as the sun, then the path of any one of them is only approximately an ellipse. In short, the old idea of *description in terms of some simple geometric curve* was inadequate. If there were only one planet, its path would be an ellipse; and, in actual fact, of all the easily defined curves, the ellipse is the closest approximation; but the other planets " perturb " it from the simplicity of movement stated by Kepler. Newton laid the foundations of the methods of calculating those perturbations, and the success attending it has been one of the greatest achievements of human

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* If the distance alters in the ratio a : b, the force alters in the ratio $b^2 : a^2$ (*not* $a^2 : b^2$).

prediction. To this end, Newton had to add a further feature to his "inverse square law," and the reader is asked to note it very carefully, as it contains that close connection between gravitation and inertia to which reference has been made.

There are times when Mars is at the same distance from the earth as the sun is, yet the force exerted on the earth by Mars is very much smaller than that exerted by the sun on such an occasion. A similar statement would be true of any other planet than Mars, were it possible to bring it to a distance from the earth equal to that of the sun. This is known from the small values of those "perturbations" from perfect ellipticity on the part of the earth's orbit, of which we have just spoken. We express this by saying that the "gravitational strength" of the sun is much greater than that of any planet; the planets also vary among themselves in this particular, Jupiter enjoying the greatest strength in the group. The third law of motion, which states the equality of action and reaction, involves another way of regarding this fact. If A is a "stronger" planet than B, then not only does A exert a greater force on a third planet C than B does, if A and C are separated by the same distance as B and C, but, *vice versa,* C exerts a greater force on A than it does on B. Newton gave an exact quantitative basis to this fact by postulating that the sun and each planet and each satellite had its own definite numerical "gravitational strength," and that the actual mutual force between two such bodies at a given distance apart could be calculated in suitable units of force by multiplying their respective strengths and dividing the product by the square of the distance. It is this complete statement of the "law of gravitation" which has permitted these strengths to be computed from the observed perturbations in the past, and thereby permitted the prediction in the future of the exact orbits to be pursued and the positions to be observed at any prescribed time. All astronomical prediction of comet appearances, eclipses, etc., have involved this simple and yet so powerful statement.

Now for the feature in question. Newton postulated that *these gravitational strengths are proportional to the inertial masses of the planets.** The usual statement of his law of gravitation—viz., that the "gravitational force between two bodies is inversely proportional to the square of their distance apart and *directly proportional to the product of their masses*"—shows this at once. Now, there is no a priori reason for this. For example, we know that the attraction or repulsion between two electrified bodies varies inversely as the square of their distance apart and directly as the product of their charges of electricity, but these charges are not necessarily proportional to the masses of the bodies. There is a similar law of magnetic force between magnetic poles, but there is no relation between pole strength and mass. The usual way of stating the law of gravitation quoted above rather conceals this important hypothesis. It is quite natural to assume that besides the separating distance there are two other factors, one connected with each body, upon which depends the mutual force, and we have called these gravitational strengths, and not masses, to begin with, just to bring out clearly the fact that a very fundamental assumption has been introduced later. Now, what evidence had Newton to offer for this step? Two very important facts. The first concerns terrestrial movement. Let us consider the earth's attraction on a small body at its surface. It is equal to the product of the gravitational strengths of the earth and the body, divided by the square of the radius of the earth.† The acceleration of the body, if it is allowed

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* The reader is implored to keep the notion of weight in the background. Such phrases as "weighing the earth" are most misleading, and confusing in this connection. "Determining the inertial mass of the earth" is the correct phrase. On what balance and over what planet would the experimenter hang the earth when weighing it?

† Newton proved that a uniform sphere of matter attracted small bodies outside it as if its matter were all concentrated at the centre.

to fall, is, by the second law of motion, obtained by dividing this force by the mass of the body. Now, suppose we choose a body having twice the gravitational strength. The force is doubled; but so also by hypothesis is the inertial mass; hence the acceleration should be the same. Now, this is precisely the case, as Galileo discovered in his famous experiments on falling bodies. Freed from air resistance, all bodies fall alike—the feather as well as the coin. The second piece of evidence which Newton brought forward was Kepler's third law of planetary movement, concerning the dimensions of orbits and the periodic times. This would not be true, assuming Newton's law of motion, unless the gravitational strengths of the planets were proportional to their inertial masses, as can be demonstrated by mathematical reasoning.

It follows, as a result of this assumption, that, whatever path one body can pursue in any "field of gravitation," can be pursued by any other body, whatever its mass and constitution in the same field, provided there is no frictional or viscous resistance. Take, for instance, a projectile. If there were no air resistance, the curved path which a stone travels along from a point A to a point B could be executed by a heavier or lighter stone *in the same time*. Note the italics. Between A and B there are an infinity of paths along which the stone can be made to travel by varying the direction and speed of projection at A, but if the time in which the journey is to be accomplished is laid down to begin with, there is only one path under the given gravitational forces, or, as we say, "in the *same* gravitational field." On the celestial scale, in the void which offers no resistance and in which inertial motion is a fundamental feature, similar facts must be true if Newton's hypothesis about gravitational strength and inertial mass be true; it is mathematically inevitable. Given two positions in the field of gravitation of the sun, for instance, there is one, and only one, path connecting them which will be traversed by a planet under the sun's attraction *in a given interval of time*. You can suppose the conditions of starting from the first position—the speed and direction of motion there—to be altered in such a way that the planet will take some other course (in this case an arc of another ellipse) to the second position, but it will be executed in a different time. *Given the time, the path is unique.*

Special emphasis has been laid on this result, because in his law of gravitation Einstein has embodied the same fact. It forms the starting-point of his investigations.

At this point the acute reader may make the following observation: " You say that Newton abandoned the heliocentric hypothesis, with space, as it were, rigidly attached to a fixed sun, just as Copernicus abandoned the geocentric hypothesis; yet implicitly all that you have talked about in the last few pages has obviously concerned motion round the sun as a centre or focus. To all intents and purpose, the sun has been fixed in our space."

The answer to this objection centres round the postulate of mechanical relativity referred to in the introductory pages. The essence of Galileo's and Newton's discoveries lay in the recognition that what is determined by the actual situation of a body in space is not its velocity, but what is happening to its velocity—*how that is changing.* The acceleration is determined by the force, which is a matter of situation with regard to other bodies, and the inertia, which is solely a matter of the body, independent, according to Newton, of all such consideration such as situation, and *independent even of the body's own speed.* (This latter is an important proviso, of which we shall take some account at a later stage.) Now, the force, being a matter of distance apart, is determined solely by that distance, and is not affected whether the sun is at rest or not, and this determines the planet's actual acceleration of velocity in space. True, what we actually observe is the planet's acceleration relative to the sun, but even if we suppose that the sun is moving with a steady speed through space, so that the planet's real velocity in space is its actual velocity round the sun, plus the sun's velocity in space, it can be shown that the acceleration of the planet's velocity relative to the sun is also its real acceleration in space, *provided velocities are "compounded" or added by the well-known parallelogram rule,* with which the reader is probably familiar. (If not, let him recall the illustration of the two cars in the Introduction, or the simple illustration of the ships travelling at ten miles per hour, while a man walks its deck at four miles per hour, and so travels over the sea at fourteen or six miles per hour, according to his direction along the deck.) This is the postulate of mechanical relativity; all mechanical occurrences "inside the solar system" go on just the same, whether the sun is at rest or in motion in the inertial void. This postulate is inherent in Newton's laws of motion. As a simple illustration, let us take any dynamical experiment which could be carried out in an ordinary laboratory; it could also be carried out, without any apparent change in its results, in a large enough vehicle travelling *uniformly along a straight line.* It is known nowadays that our solar system is in motion with reference to the system of stars, and is travelling towards a point in the constellation Hercules. The motion is probably orbital, but its sweep is so large that for all practical purposes it is uniform for ordinary lapses of time. It is also known that the stars are in relative motion to each other, but so distant are they from our solar system that it has taken many years of careful observation with precise instruments to substantiate this fact and measure the minute angular changes involved. Furthermore, Newton's law of gravitation has been shown to be operative in the case of some binary stars. Thus the array of evidence for Newton's laws, and his postulate of an absolute space, is pretty formidable. Let us consider this idea and its concomitant, the idea of absolute time, a little more closely.

CHAPTER III

ABSOLUTE SPACE AND TIME

Some attempt has been made in the previous pages to outline the complete change in outlook between those days when men thought of themselves as the centre of all things, the earth being the one fixed immovable thing, and space only a kind of geometrical partition between bodies, and these when men think of space as a great infinite stretch of "nothingness," which, nevertheless, could exist without matter, of which matter "occupies" small portions and in which it moves.

The close agreement of the observed facts with the deductions from the laws of motion, which embody the principle of unchanging inertia or mass for all matter, and the law of gravitation, which embodies the unchanging gravitating power and its close relation to inertial quantity, have rendered Newton's ideas unchallengeable for two centuries. Whatever the philosophers had to say about his philosophy, they had to accept his physical science, and that rendered any attack on his cosmological views practically hopeless. Of course, one must not lose sight of the fact that the validity of his laws in an absolute space, as distinct from their validity in a relative sense within, say, the solar system, rests on the truth of the postulate of mechanical relativity, which, in its turn, is closely bound up with the assumption that the results of Euclidean geometry are true in this absolute space, and with Newton's conception of an absolute time.

Inasmuch as the average person never pursues his studies in geometry beyond his school years, there is little general realisation of the nature of the validity enjoyed by Euclid's reasoning, on which all school geometries are based. Euclidean geometry is entirely *theoretical.* It starts out from certain axioms and postulates—*i.e., assumptions,* and deduces certain conclusions. As to the physical truth—*i.e.,* the truth in natural fact, of these conclusions, that is entirely a matter of *experimental* geometry, which enters into practically all physical experimenting, into surveying and geodesy, and into observational astronomy. In these processes lengths are measures—*i.e.,* distances between marks in *accessible* places, and other lengths, too minute or too large to be measured directly, or distances between inaccessible spots, are *inferred* from these by the application of mathematical processes, many of which assume the truth of Euclid's results. It is, of course, a matter of common knowledge that the consistency and agreement of these inferences is so good that they offer no challenge to the postulated truth of the assumptions on which the theoretical geometry is based. Nevertheless, the real state of affairs should not on that account be overlooked.

Take, for example, the concepts of point and line. They are really undefinable in the strict sense. The straight edge of a ruler, or the pencil stroke on a piece of paper, are not " lines " in the geometrical sense. The line is an abstraction of the mind, and we assist the immature mind to reason about these abstractions by presenting visual pictures to it. Of course, you cannot reason about a creation of your imagination unless you admit that it possesses a few unchanging properties, and Euclid requires us to admit that no two *straight* lines can cross at more than *one* point. That is entirely reasonable, as it conforms so obviously to the behaviour of pencil strokes which we draw along rulers, but we must observe that that is the only requirement which Euclid imposes on the notion of " straightness." In other words, an *enclosed* area cannot be bounded completely by only two straight lines. You may have a triangle formed by straight lines, but not a " diangle " or two-sided figure. Most of the other axioms which Euclid introduced into deductive geometry are of such a nature that to deny them would seem to destroy the possibility of any reasoning at all. But there is one axiom which has caused an enormous amount of argument; it concerns the possibility of straight lines in one plane not cutting at all.

Euclid admitted that possibility, but limited it in such a manner that his postulate amounts to assuming that given one straight line in a plane and a point in the plane, but not on that line, there is *only one* straight line in the plane through that point which will not cut the first one anywhere " fore or aft." The average man regards this as so " obvious " that he is bewildered when it is questioned; for he recalls the school classroom and his struggles with theorems on parallelograms, etc., and he makes pencil marks " in the same direction " on a piece of paper and waves his pencil in an airy way into the vasty deeps of space to show the foolishness of any doubting Thomas. But surely the reader has never produced his *pencil marks* beyond the edge of the drawing-paper, and yet the axiom says distinctly " no matter how far produced in either direction." Even let us admit the removal of that restriction and give him as smooth a drawing surface as possible, extending to any distance over the earth's surface, and let him produce his two lines on and on. Well, every pair of *straight* strokes will meet somewhere. " No," says the objector, " the strokes which the draughtsman started with ' looked ' straight because they were short, but as you go on you see that they are arcs of great circles on the earth's surface, and are not straight lines at all." Well and good, we must produce " into space." But who has done it and can tell us what happens to material strokes or tight threads or whatnot, let alone what happens to the mental abstractions based on such things?

In putting forward these ideas, one is not questioning the " truth " of Euclid's results. Granting his axioms and postulates, the conclusions are irrefragable. But you can deduce other results just as " true " in the same sense if you start from other axioms. *E.g.*, from Euclid's parallel axiom you can deduce that the three angles of a triangle make up two right angles. If, on the other hand, you assume that *all* lines in a plane intersect somewhere—*i.e.*, that no parallel lines exist, you can deduce that the three angles of a triangle are together greater than two right angles, the excess being greater the larger the area of the triangle. Or, if you like to assume that through a given point you can draw more than one line in a plane which will not meet anywhere a given line in the plane, you can prove that the sum of the three angles of a triangle is less than two right angles, the defect being greater the larger the area of the triangle. Now which of these three deductions is *physically* true? It must be admitted that the evidence of mensuration is all in favour of Euclid. Yet it does not preclude the possibility that mensuration of triangles on a celestial scale, instead of terrestrial, were it possible with sufficient accuracy, might reveal a discrepancy which could not be treated as an "experimental error." In any case, the reader must accommodate his thoughts to the possibility; for here lies the clue to that phrase "curved space," which seems so peculiar and meaningless to many, and which is such a feature of popular explanations of Einstein's views. If Euclid's axiom is assumed to be true, a triangle can be drawn on a plane with straight lines, and its angle-sum is equal to two right angles. On a curved surface you cannot draw a triangle with straight lines, but you can do so with "geodetics" ("Geodetic" is the geometer's name for the line stretching along the path of shortest distance between two points on the surface). This geodetic triangle has an angle-sum which is not, in general, equal to two right-angles, being greater for certain types of surface, and less for other types. Thus, if Euclid's axiom is *not* assumed, the properties of the rectilinear triangles drawn on a plane are similar to the properties deduced from Euclid's axiom for geodetic triangles on a curved surface, or, more briefly, the non-Euclidean geometry of the rectilinear triangle on a plane is similar to the Euclidean geometry of the geodetic triangle on a curved surface. So if it turned out that *as a physical fact* the angle-sum of a triangle with three stars as vertices was not exactly two right angles, we could take our choice of two statements: either "space is flat and physical geometry is non-Euclidean," or "space is curved and physical geometry is Euclidean." In any case, the reader is not asked to perform the impossible feat of visualising intuitively "curved nothing," he is merely reminded that when he meets the phrase "curved space" he is to think that he must be wary about the too easy assumption that the conclusions concerning geometrical figures in Euclid's theorems are *of necessity* valid concerning the *material frames,* in Nature of which these figures are mental abstractions.

In the sense indicated, Newton assumed that his absolute space was "flat" and its geometry Euclidean. But dynamics involves more than geometry; it involves the conception of changing configuration, not mere fixed situation—*i.e.,* the idea of time enters. Everyone has that intuitive perception of "duration" in events, by which we make rough and ready estimates of lapse of time. This perception of duration is the analogue of our perceptions of extension of matter in space. Just as we make our mental abstractions of point, line, and plane from the latter, so we conceive instants as "points in time" with no duration. In this way we arrive at the notion of physical time as something "flowing uniformly," independent of all natural occurrences and all sense-impressions on our part. This was another of Newton's fundamental postulates, first clearly formulated in his *Principia.* Just as the distance of a point on a line from another point on it could be quantitatively stated by a precise number and unit, so the interval between one instant and another could also be quantitatively stated by a precise number and unit. When he speaks of rate of change of velocity and momentum in his writings, he means the quotient of a change of velocity or momentum between two states of a body by the interval of this time between these states. If the reader asks how Newton could postulate "uniform flow" of anything without reference to something else, he is reminded that this assumption is just the feature of Newton's *absolute* time, as distinct from the conception of a time relative to an observer's situation and environment, which is the feature of Einstein's work. But he must not rush too hurriedly to the conclusion that this assumption was entirely metaphysical. It was not introduced, as it were, *in vacuo;* it was introduced as an accompaniment to his laws of motion. If inertial motion be a feature of absolute space, then on observing a body far enough removed from the gravitational influence of other bodies, one would have a "timepiece"; for the distance it travelled would be a proportional measure of time lapse. True, we cannot observe the absolute movement in space, so that this procedure for measuring time, though theoretically conceivable, is not merely impracticable, it is impossible. But that does not dispose of all the possibilities.

It can be proved from Newton's laws that provided the resultant force on a body like the earth passes through its centre of mass, then it will rotate uniformly in the absolute space, so that if we observed the angles turned through by the earth, we would have a practical measure of his uniformly flowing time. It was to this fact that Newton turned for a justification of his ideas. He was able to show that provided his law of gravitation held between the smallest parts of matter, then the resultant force on the earth, due to the gravitational action on it of the matter in sun and planets, had a line of action passing so near to its centre of mass, that the discrepancy from uniform rotation, due to this or to tidal friction, was so slight that this rotation would, indeed, be an excellent measure of time. "But," says the objector, "you cannot observe rotatory motion in an absolute space any more than you can observe translatory motion." To this statement Newton would have given an unqualified denial, and it must be said there is some powerful evidence in physical experiment in his favour. True, the rotation of the earth on its axis usually thought of is relative to the fixed star frame, and it is this rotation which is certainly the basis of all our measurement of time. Nevertheless, Newton pointed out that the bulging at the earth's equator could hardly be regarded as an effect due to the rotation of the earth relative to the fixed stars.

Such a gravitational effect *due to relative movement*, even if postulated, could hardly be regarded as sufficiently large, considering the distances involved. He was also able to point to other phenomena, into which there is no opportunity to enter; but since his time, a very famous experiment has been performed, which appears to justify him to the hilt. This was first tried about the middle of last century by Foucault in Paris. A long pendulum, suspended in such a way that it was free to swing in any vertical plane, was made to vibrate, being very carefully started in some definite vertical plane. As the day wore on, this plane of vibration altered at a perfectly definite rate, so that to a person looking down from above, the line of motion of the bob just over the floor turned round in a manner similar to the hands of a clock, taking about five and a half minutes to turn one degree. The theoretical investigation showed that at the Equator no such phenomenon would have occurred, while at the North Pole the rotation of the plane of swing would have been faster, one degree only occupying four minutes, and subsequent repetitions of the experiment in other latitudes abundantly support these conclusions. Now the hitherto accepted explanation of this experiment (conceived for simplicity as being performed at the North Pole) is that since there is no component of the earth's attraction on the bob perpendicular to the plane of the swing (the force being vertical and in the plane of the swing), the plane of the swing is not turning at all, *but maintaining the same orientation in absolute space*, and that what we perceive is the turning of the building around the earth's axis, the plane of the swing giving us a definite direction in space to go by.* This experiment

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* Of course, the plane of the swing may be moving in space. The point of this explanation is that it remains parallel to itself; it does not really turn round. This assumes the North or South Pole as the place of experiment. Elsewhere the matter is a little more complicated, but equally powerful as evidence for absolute rotation.

was accepted as a clinching argument for the validity of the idea of absolute rotation. There was no question of stars entering into the explanation. The Foucault pendulum was observable if an impenetrable cloud-bank had for ever shut the stars from our vision. But it turned out that the rate of rotation, as observed from Foucault's pendulum, was, within experimental error, the same as the rotation with reference to the fixed star frame, and this was taken to mean that the stars, although not fixed in absolute space, were moving in it at a rate sufficiently small to give, considering their enormous distance from us, a material frame of reference, which could for some purposes approximately replace absolute space. To sum up the situation: translatory motion of the earth relative to the fixed star frame has been observed (we and the constellation Hercules are certainly approaching one another), but translatory motion in an absolute space has not been observed; there does not exist any *measured* speed which, even on any plausible grounds, can be called an absolute speed of the earth in space. On the other hand, rotatory motion of the earth relative to the fixed star frame has been observed, and, in addition, a *measured* rotatory motion, quite independent of this, to which there are considerable grounds for giving the name "absolute rotation."

Returning to the considerations of time measurement, which necessitated this digression, the earth rotating in absolute space was Newton's fundamental clock. Theoretically, even this was not precisely accurate for measuring his "absolute time," unless the whole gravitational force on the earth passed accurately through the centre of mass; but even so, any discrepancy arising from this would not be serious, and could be calculated and allowed for by astronomers after observation over a sufficient number of years. The rotation in absolute space* was not directly measurable in Newton's time, and even now,

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* *I.e.*, as measured by Foucault's pendulum or other similar devices.

when we think that it is so, it is best, from the point of view of exactitude of observation, to take the "sidereal day," the time of rotation relative to the fixed stars, as our practical unit. As pointed out, the two measured rotations are, at all events, so near in value as to render this procedure justifiable. Added to this there is another justification. There is a well-known result, deducible from Newton's laws of dynamics, showing how to calculate the period of swing of a pendulum from its length and the intensity of gravity at the place. Now, strictly, this period is an interval of Newton's absolute time, and the formula implies a constant ratio between the period of the pendulum and the period of one absolute rotation of the earth. As a matter of experiment, there is a constant ratio between the period of a pendulum and the period of rotation of the earth relative to the stars. This, in fact, is the justification of our use of pendulum clocks as practical instruments for measuring time.

In this way Newton projected all the conceptual apparatus required for dealing scientifically with terrestrial movements into the celestial world outside. So lucid, succinct, and powerful were the brief statements expressing his laws in words, so easy of transformation into the mathematical dress required for calculation and deduction, and so close the conclusions to the facts that it is little wonder the civilised world has regarded his achievements as supreme in their own sphere.

But in his own lifetime there occurred the first successful attempt at an experiment, which carried in it the germ of those facts which have shown that even this great man had not reached the absolute truth. Many people had attempted to find whether the transmission of light was instantaneous or not. Galileo was one. None succeeded, until about 1670 a young Dane, Olaf Römer, engaged at Paris on astronomical observation, pointed out that a peculiar feature of the eclipses of Jupiter's satellites could only be accounted for by the assumption that the light from the satellites took several minutes to cross the earth's orbit round the sun. Little notice seems to have been given to this result, on account of the conflicting values for the time deduced from observation of different satellites. More than half a century later, however, the English astronomer Bradley produced supporting evidence from other astronomical facts, to which reference will be made presently, and, in the meantime, better observation had cleared up the discrepancies in Römer's calculations, so that just about the time of Newton's death in 1727, the gradual propagation of light was an accepted physical fact, and the time of transmission across the earth's orbit was determined to be sixteen minutes twenty-six seconds, which is only about ten seconds too little, according to our present-day knowledge.

Now it may be said that around this fact of the gradual propagation of light and its consequences have clustered all those doubts and confusions which have entailed within the past two decades that recasting of our views connected with Einstein's name. Let us devote a short chapter to these disturbing *optical* phenomena.

CHAPTER IV

THE OPTICAL PHENOMENA

Our historical outline now moves on to the early years of the nineteenth century, when the undulatory theory of light was beginning to receive the serious attention which it had missed in the previous century owing to the great fame of Newton, who gave his support to the corpuscular theory, on what were at the time very sound reasons. The conception of a luminiferous ether, as a medium endowed with definite material properties as regards elasticity and density, began to emerge in the writings of the foremost physicists. To a brilliant young French engineer, Augustin Fresnel, is due the credit of the first success in framing a dynamical theory for the transmission of light regarded as an undulation and not a stream of discrete particles. By degrees, absolute space began to assume an air of "substantiality." True, this medium had to be endowed with some strange properties; it had to possess an elasticity enormously out of proportion to its density by comparison with ordinary matter (that was by reason of the tremendous speed of propagation of light vibrations through it); and it had, at the same time, to display a fluidity so perfect that planets could move through it with no obvious resistance. Still, this troubled no one seriously; for, after all, the ether was not "ordinary matter," and there seemed to be no à priori reason why the limitations known to be true of matter should necessarily be imposed on the ether. The initial successes enjoyed by this view in explaining the phenomena of interference, diffraction, and polarisation of light were so considerable that the mathematical formulation of the "elastic solid theory of the ether" has been justly regarded as one of the chief scientific triumphs of the nineteenth century. Quite early in its career we meet the question of possible effects produced by the earth's "drift" through the ether, as for various reasons it began to be accepted that this ether was the "substantial thing" by which absolute space could be identified. The ether was the "absolute fixture." To help the reader to grasp one or two essential ideas in this connection, imagine a very simple occurrence. A boat is at rest in water, and over the side rests a wet oar, from which there falls a regular succession of drops; each drop starts an expanding ripple, and the concentric ripples travel out with a definite speed, the distance along a radius from ripple to ripple remaining the same. Now, let the boat move slowly through the water. The centre of one ripple is not coincident with that of the previous one, but separated from it by the distance travelled by the boat during the period between drops. But each ripple expands as before at the same speed; the velocity of the ripple *through the water* is not affected by the speed of the boat. In a similar way it was part of the accepted theory of light that the speed of light *through the ether* was not in the least affected by the motion of the body from which the light emanated. Returning to the water analogy, if a body floating in the water were at rest in it, the ripples would travel past it at their speed through the water, but if the body were in motion, the ripples would not travel past it at that rate; this might be as great as the sum of the ripple speed and the body's speed, if ripples and body were moving opposite to one another, or as small as the difference, if travelling through the water in the same direction; if the body's velocity were neither codirectional with nor opposite to that of the ripples, the *relative* velocity of the ripples past the body would have a magnitude intermediate between the former limits, *and its direction would also be changed;* thus if, looking towards the boat, the body were travelling to the right, the ripples would seem to pass the body as if coming from a source somewhat to the right of the boat. Here emerges the postulate of mechanical relativity in this connection; for while the speed of the source was regarded as having no influence on the speed of light through the ether, yet it was regarded as inevitable that the speed of the earth through the ether should have an effect on the speed and direction of the light received by an observer on its surface—*i.e., relative to him.* Now, as regards the speed, there was no direct evidence one way or the other. Michelson's refined measurements were still to come, but as regards direction, there was *apparently* very decisive evidence in the so-called annual aberration of the stars, the discovery of Bradley referred to in the last chapter, by which he was able to verify Römer's work. At all events, the changing direction of motion of the earth in its orbit round the sun did produce a constant change of direction (small, no doubt, but perfectly measurable) in the line along which one had to point a telescope at a star each night at a definite *sidereal* time. This was explained as due to the fact that the earth's motion through the ether, being the resultant of the sun's motion through the ether and the earth's motion relative to the sun, experienced a gradual change of direction throughout a year, and so caused a variation in the apparent direction of the light from a distant and approximately fixed source, which would oscillate between extremes in the course of the year.

But now let us turn to another effect on light which might be expected on any undulatory hypothesis as arising either from the motion of the source or from the motion of the receiver. If the boat is moving, the ripples are no longer concentric, they expand from different centres. In the direction of the boat's motion they get "crowded up" a little, the distance from crest to crest is reduced by the amount the boat travels in the interval between drops; in the opposite direction they are separated further apart by an equal amount, and in other directions we get crowding or further separation of smaller amounts. Thus, a body floating on the water, which would be made to rise and fall by the ripples, would do so more frequently if in front of the boat and less frequently if behind. Reasoning by analogy, if the earth were fixed in space and a star moving, the light from the radiating mechanism in the atoms of a definite chemical element in the star would not have quite the same frequency as if the star were also at rest—*i.e.*, it would not have the same "wave-length," and the reader is probably aware that the physicist possesses in the spectroscope an instrument of great precision for observation and measurement of the wave-lengths of the different qualities of lights which produce the various "lines" in the spectra of incandescent or electrically excited elements. This anticipated difference in the wave-length of light from an element in a star and the same element on the earth actually exists, and is called the "Döppler effect." But it would also be produced if the star were at rest and the earth were in motion; for in the analogy, if the boat were at rest and the ripples expanding from a common centre, a body moving towards the boat would rise and fall more frequently than if also at rest, while if moving away from the boat it would oscillate up and down less frequently. The same thing would happen if both the boat and the body were moving through the water, provided there was relative motion—*i.e.*, provided one was separating from or drawing nearer to the other. In short, the Döppler effect is evidence of *relative* motion of the earth and star, but without some other evidence as to the magnitude and direction of the earth's velocity one cannot separate the observed effect into two parts and say that this part is due to the real motion of the star, this to the earth's motion.

The truth is that neither the annual aberration of the observed star positions from where they would be seen if the earth were not revolving round the sun, nor the Döppler effect gives any direct evidence of the earth's absolute motion at all. They are phenomena of relative motion, and any suggested analysis of them is futile unless a direct measure of the earth's motion through the ether is made somehow. In connection with aberration, it may be interesting to refer to an experiment whose result has some significance in view of the recent developments. It was suggested by Boscovich some years after the discovery of annual aberration by Bradley, that if a telescope were filled by water, the angle of aberration (which is observed by a slight progressive change in the axis of the telescope sighting a given star at definite sidereal time each day) would be changed, since the ratio of the speed of the earth to the speed of the light *through the telescope* (which is the determining factor in the matter) would be changed owing to the altered speed of light through the water. It was before the days when the velocity of light through water, glass, etc., had been measured by terrestrial experiments, and it was Boscovich's idea that such an experiment would settle whether the speed through water was faster or slower, a question of the utmost importance at the time, whose answer (not obtained until nearly a century later, and then by quite other means) would prove decisive as between corpuscular and undulatory views of light. For various reasons, mostly concerned with technical difficulties, Boscovich's idea was never put into practice until 1871, when Airy, then Astronomer Royal of England, actually tried it after adapting, with considerable risk of damage, a large telescope at Greenwich for the purpose. In the meantime, however, Fresnel, about 1820, had, subjected the suggestion to a critical discussion in the light of his own recently formed theory as to the "elastic-solid ether," and had shown that Boscovich's mathematical proof was too inadequate for the complexities of the relations between ether and matter, and actually predicted that, although there was a difference, it would be too minute to be observed in practice. As a matter of fact, this is just what Airy found, and as things stood then the result was triumphant justification of Fresnel's acute criticism. There would be no point in telling this interesting little romance of physical science here, but for the fact that it will prove significant at a later stage.

Our narrative has now reached the period mentioned in the introduction. It became apparent that if any successful attempt were to be made on this elusive absolute velocity of the earth, it must be done by "direct assault." A change in the velocity of light must be actually found, and the genius of Michelson devised an instrument (known as "Michelson's interferometer") sufficiently precise to do so. His source of light was a terrestrial one. As stated above, the velocity of the *source* through the ether would, on current views, have no effect on the velocity of light through the ether, and, as a matter of fact, there is pretty decisive astronomical evidence against any assumption that it would have an effect. But the velocity of the receiving apparatus through the ether would have an effect on the *observed* velocity, which would be, on traditional theory, the resultant of the true velocity of light and the reversed velocity of the earth (*i.e.*, the receiving apparatus) through the ether. Both source and receiver, being so near, would have the same velocity through space and therefore no *relative* velocity to each other, and so any observed effect could not be ascribed to *relative* velocity, as there was none.

As already stated in the introduction, no effect was observed at all. Michelson, in a sense, timed a race between two beams of light, each one travelling along a path to a distant mirror and returning along its path to the common starting-point. One path was at right angles to the other. The decisive quantity was not the times of the journeys made by each beam, but their difference, the amount by which one beat the other. As the apparatus was slowly turned round, this difference should have *changed,* the winner winning by less and less, and then losing by more and more through one half-turn, the process being reversed in the second half-turn. No such thing happened.

Here, then, was the situation. No effect, mechanical or optical, had ever been definitely traced down to the *absolute* velocity of the earth through space.* Effects due to velocity had certainly been measured, but the effects were only there when there was *relative* velocity. If there were no relative velocity, as in Michelson's experiment, there was no effect. Even so, this feature of the situation did not seem to strike anyone as important; and there followed the period of twenty years referred to in the introduction, during which one attempt after another was made to bring in the absolute movement of the earth as a relevant feature of natural phenomena, culminating in the work of Lorentz. As the last stage of our pre-Relativity journey, some attempt will be made in the next chapter to indicate the idea of Lorentz's solution. It will carry with it also some idea of a change which had in the meantime come over the attitude of the physicist to the theory of light propagation and, indeed, to the theory of the constitution of matter itself, without which Lorentz's solution, let alone Einstein's Relativity Hypothesis, would not have been possible.

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* The Foucault Pendulum result was attributed to an absolute *rotation* in space.

CHAPTER V

THE ELECTROMAGNETIC THEORY OF LIGHT AND MATTER, AND LORENTZ'S SOLUTION OF THE ABSOLUTE MOTION DIFFICULTY

THE reader will recall from the Introduction that the difficulty raised by Fitzgerald's hypothesis of contraction (which explained the Michelson experiment) centred round the inability to discover those effects on the electrical and optical properties of matter which, it was *inferred,* should accompany such a contraction. To Lorentz belongs the credit of showing that the inference was not justifiable, and that the absence of these effects was not so fatal to the hypothesis as appeared. A number of essays on this point were written by him towards the close of the nineteenth century, and in 1904 he summarised his work in a classical paper to the Academy of Sciences at Amsterdam. In these papers one hears no more, however, of the " elastic-solid " ether, for Lorentz's life has extended over that period during which a remarkable change came over the scientific view as to the nature of light. It was still regarded as an undulation, but in what exactly the undulation consisted was not so precisely defined *in terms of matter and motion* as before. Shortly after the middle of the nineteenth century, Clerk-Maxwell, the first Professor of Physics at the Cavendish Laboratory, Cambridge, had developed in a theoretical manner the ideas of Faraday about electricity and magnetism to such a point that he was able to predict that " waves of electric and magnetic force," or " electromagnetic waves," should be capable of propagation through space. No one in these days of universal broadcasting requires any proof of the perfect soundness of that " forecast." But nearly twenty years elapsed between Maxwell's mathematical proof and Hertz's actual production of the first " wireless waves " in his laboratory at Munich, an event which took place about the same time as Michelson's first experiments on the "ether-drift." So people took little notice of "Maxwell's equations" for many years, and still less of a very far-reaching hypothesis which he founded on them—viz., that light was just such a transmission of electric and magnetic forces. People were too "dynamical"; they wanted a material ether, something almost palpable which could "wobble." There was for their minds something more satisfying in the picture of a "quivering ether" than in a vague, unsubstantial "oscillation of electric and magnetic force"; and, after all, where were these electromagnetic waves? Maxwell had predicted them, no doubt, but he had given no experimental justification. The latter criticism had no weight after Hertz's experiments, and by degrees a subtle, almost unconscious, change crept over the minds of scientific men, especially the younger generation, regarding light. Less and less did they worry about a "model ether" with definite properties as regards elasticity and density; its mechanical trappings fell away from it by degrees. At first even those who admitted the power of Maxwell's theory felt hardly at home with it unless they could conjure up to their mind's eye some picture of the ether—in filaments and vortices, for example—whose motions of translation or rotation, or whose quiverings were the underlying reality of the electric and magnetic forces of the waves. Therein we see the waning influence of the purely mechanistic view of the nineteenth century. Many of our leading physicists whose early training took place in the middle and later years of that century could never quite free themselves from such prepossessions (a perfectly natural result, and one concerning which only ignorance could utter any gibe). But slowly the leaven of Maxwell's idea worked, so that nowadays we are so far from worrying about a material ether as a transmitter of a mechanical radiation that we actually regard radiation as an entity in itself, requiring no medium for its transmission, having an independent existence quite as real as that of an atom or electron. This change of attitude, coupled with the discovery that the atoms of the chemical elements (hitherto believed to be indivisible) were themselves constructed from still smaller parts, reacted on the scientific view as to the nature of the forces exerted between atoms and molecules. We all know that the antenna of a broadcasting station radiates " waves " which can travel at an enormous speed and can excite oscillations of electricity in an aerial wire. These oscillations we " receive " and amplify for our own purposes by means of " valves, coils, and condensers." Now, the electromagnetic waves which constitute light have to be radiated from something and " received " by something. The radiators are the atoms of matter in the luminous body, or rather those smaller parts—nuclei and electrons—which are really minute portions of positive and negative electricity whose movements within the atom give rise to the waves. Each atom is a minute " broadcasting station," emitting electromagnetic waves, each type of atom having its own characteristic wave-lengths or frequencies. Just as the broadcasting stations of the world emit as a whole waves whose wave-lengths extend over a range from a few metres to several thousands of metres, so the various chemical atoms and molecules emit waves having a range from about one ten thousandth of a micron* to several hundred microns, or nearly a millimetre. The waves are on a much " tinier " scale, but essentially similar in nature. We also know nowadays that with a given coil and condenser we can " tune " our receiving-sets to detect any wave whose wave-length is in a certain range or " wave-band " (say 300 to 500 metres). The atoms in our eyes are naturally tuned to detect waves from about ·4 to ·8 of a micron, and we use photography and various ingenious instruments to detect the shorter and longer waves with which our eyes cannot deal. But these structural details of the atom which give rise to radiation are, we now believe,

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* A micron, or micrometre, is one millionth of a metre.

also responsible for those forces of "affinity and cohesion" (to use the old terms) which bind the atoms in the molecule, the molecules in the solid body, as well as the electrons and nuclei in the atom. So there is an electromagnetic origin for the forces exhibited in the constitution of matter as well as in the propagation of radiation.

This, then, was the kind of new conceptual material with which Lorentz was tackling this old problem. Purely mechanical considerations are not very pronounced in his writings; one reads nothing about density and elasticity of an ether; the ether has sunk to the level of some vague kind of stuff filling space, with regard to which light has a certain speed. Speed relative to "space"—*i.e.*, "emptiness"—cannot exist; we must invoke some "material" for that purpose, even if it has no other relevant connection with the problem. So far, at all events, had we travelled on the road to Relativity. By the aid of Maxwell's equations, or rather a generalisation of them due to himself, Lorentz proceeds to discover the relations between the forces exerted between the particles of a body when in motion "through the ether" and those which exist when the body is at rest. His prime discovery was that if he assumed *that the ultimate electrical parts of the atoms, the electrons, were subject to a Fitzgerald contraction, when in movement through the ether*, two results follow: (1) All the particles, atoms, and molecules of the body in motion draw closer together, owing to the changes in the intermolecular and interatomic forces, in just the same proportion, so that the body as a whole suffers a Fitzgerald contraction of the same amount as the individual electrons; (2) the relation between the forces existing in the body at rest and those in the body in motion takes a special form, from which it can be inferred at once that all those effects which were expected to follow from the contraction *in bulk of the body as a whole* ought actually to be absent.

What the reader should observe is that the "Lorentz contraction hypothesis" is something more fundamental than the "Fitzgerald contraction hypothesis." The latter was a postulate about matter *in bulk*, which raised as many difficulties as it settled. The former was a postulate about the ultimate electron, which carried the contraction in bulk as a natural result, but which also cleared away those difficulties which the latter, as a bald *ad hoc* hypothesis, had given rise to. It should also be noted that Lorentz's contraction hypothesis was not merely *ad hoc;* it had considerable support from the experimental study of the electron itself in a manner which ought to be presented to the reader, inasmuch as it shows another remarkable change of attitude on the part of the physicist.

It will be remembered that the central doctrine of the Newtonian theory is the *invariability of mass.* The mass of a body (*not* the "weight," which depends on situation as well as on mass) does not change, whatever changes may take place in size, temperature, etc.; in particular it does not depend on *velocity*. The same force will produce the same acceleration, if exerted on the body when at rest, as if exerted on it travelling with any speed we like. Now, as far back as 1882, Sir J. J. Thomson discovered in a theoretical discussion based on Maxwell's equations that if a body were electrified when in motion, its inertia should be greater than if unelectrified; furthermore, that this "extra mass" *is not invariable, but should increase with speed*. No one at that time realised the import of this small cloud rising on the horizon. This "electromagnetic inertia" (as it was called to distinguish it from the "ordinary inertia") was attributed to moving "Faraday tubes" ("tubes of ether," if you like) attached to the electricity on the body, and Newtonian invariability was formally satisfied. But little by little the doubt grew. "Ethereal tubes" have gone the way of "elastic solid ethers." When the electron, the *permanently* electrified part of the atom, was isolated and studied (first by Sir J. J. Thomson himself), it was naturally asked: (1) "Does the electron's mass increase with velocity?" (2) "If so, how much of its mass is ordinary and invariable, and how much electromagnetic and variable?" The answer of experiment was: (1) "Yes." (2) "All the mass is electromagnetic." This was so important a matter that no stone was left unturned to attain the utmost precision in experiment. Probably no prime investigation in physical science has received so much attention from first-rate experimentalists, with a resulting certainty about the data which leaves nothing to be desired. Now, the beauty of Lorentz's hypothesis about the contraction of the electron was that by aid of it he was able to show that the increase of electron mass accompanying increase of velocity agreed remarkably well with the data of experiment, much better than did the increases calculated from other views as to the behaviour of the electron, current at the time as alternatives to Lorentz's. Of course, the reader should realise that this increase of mass is too small to be experimentally perceived for ordinary terrestrial speeds or even stellar speeds. In electric discharge tubes (such as X-ray tubes) we are, however, dealing with atoms and subatomic particles flying at speeds from one tenth up to almost the speed of light itself, with a wealth of data about these matters naturally not known to our predecessors. It may be of interest to state the matter precisely. Supposing v to represent the electron's speed through the ether, and c to represent the speed of light, then Lorentz's hypothesis was that the dimensions of the electron parallel to the direction of motion contracted in the ratio

$$1 : \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

This entails as a result that the mass should increase in the ratio

$$\sqrt{\left(1 - \frac{v^2}{c^2}\right)} : 1$$

and this result is certainly verified in experiments on electrons in discharge tubes where v becomes comparable in value with c. When v refers to the comparatively slow motion of a body on the earth through the ether, the changes in size and mass of the electrons in the body are, of course, very minute; nevertheless, they are theoretically capable of accounting for a similar contraction in bulk (Fitzgerald contraction), and a consequent explanation of the Michelson experiment, and for the absence of those effects which were originally thought to follow as a matter of course from this bulk contraction.

In a book on Relativity it may seem remarkable that the author has finished three parts of the volume before beginning a formal explanation of what that word means. He justifies this procedure, to himself at all events, by the experience of academic and popular lectures endeavouring to make clear these difficult matters; for he believes that no one, even a skilful mathematician, can grasp the meaning of Einstein's theory unless he has more than mere unco-ordinated and hazy notions of the ideas of space, time, matter, gravitation, and light which have arisen in the past and gone their way after a period of usefulness. Equally he believes that Relativity could not have arisen apart from the electromagnetic theories of light and matter, and so anyone hoping to understand it should have some idea of the results of the impact on the traditional materialism of the nineteenth century, made by the work of one of its greatest physicists, Clerk Maxwell. The author therefore makes no apology for devoting nearly fifty pages to these topics. If anything, it is too little, not too much.

Alas for the vanity of human hopes! Just when at last this age-long problem of absolute space seemed to have attained to the condition of happy solution, the young Einstein of 1905 pointed out a peculiar feature of Lorentz's work which had escaped notice. Lorentz had set up a comparison between a body at rest in the ether and a similar body in movement through it. The only way in which the "ether" came in was in the particular mathematical dress given to the Maxwell equations as applied to the first body, and the somewhat different dress as applied to the second. What Einstein noticed was that in reality the symbol representing the velocity of the second body "through the ether" could just as readily be interpreted as the velocity of the second body relative to the first; further, if one chose to consider the second body at rest and the first body moving, using the same symbol, *with a minus sign before it because of reversed direction of motion,* to represent the velocity of the first body relative to the second, absolutely the same mathematical results followed. In short, the mathematical relations give not the slightest clue towards deciding which body is at rest in the ether and which is moving through it. "Naturally so," is Einstein's comment. We have worried ourselves for generations with an idea which may have value on *metaphysical* grounds, but has no relevance at all for physicists. Not a single experiment in any branch of science has given direct evidence of the existence of a translatory motion in an absolute space, although we have numerous experiments to show a whole variety of effects arising from relative motion of matter to matter. Why, therefore, should we postulate the existence of motion in an absolute space and then expect the theorist to devote such skill and ingenuity in devising proofs that we can't observe it—*i.e.,* that it does not really *exist* for us at all. Let us be *experimental* physicists first, and begin our theoretical reasoning on postulates directly related to the fundamental fact that absolute motion has no significance in physical science, while relative motion has. When Einstein began in 1905 to consider the repercussions of this idea on our notions of space, time, inertia, light, and electromagnetism, he found that the postulate that the forces involved in the constitution of matter are of electromagnetic origin was entirely consistent with his views. One phenomenon alone appeared to stand outside—gravitation—*if Newton's law were absolutely true.* This difficulty he ultimately conquered by discovering a law of gravitation which is just a little more accurate than Newton's, and which, by completely abolishing the notion of gravitation as an "action at a distance," makes its absorption into the Relativity scheme quite feasible. This was about 1915. So the Relativity of 1905-1915 is generally referred to as the "Restricted" or "Special" theory, the later Relativity as the "General" theory.

CHAPTER VI

THE SPECIAL THEORY OF RELATIVITY

Einstein's two postulates are:

1. Absolute motion has no observable effect on any physical phenomenon—*i.e.*, it does not exist physically.
2. The speed of light is the same in all directions at a given place, and its value at one place is the same as at any other place in the universe.

The second postulate was based on the results of the Michelson-Morley experiment; the first on the utter failure to observe effects due to a velocity which was not a velocity relative to some body, or, as we say, velocity in a *materially defined* "frame of reference" or "space." Naturally, all those experiments which sought for effects due to the supposed Fitzgerald contraction come under the latter heading, and this ended our worries about them. We no longer seek to "explain" them. It *is so*, and we frame our fundamental principles accordingly.

Now it is well known that these postulates, although they certainly bring simplicity and precision into matters where there previously existed complexity and doubt, do abolish the possibility of a universal time-order. When Newton introduced the notion of absolute time into scientific thought, he did not consider the time involved in the transmission of effects from one part of the universe to another. He regarded gravitation as transmitted instantaneously; only in his own lifetime was the discovery made that light required time for transmission. Naturally, he would have no idea what the ultimate effect of this on our views would be. However, this notion of an absolute time is so woven into the texture of our ordinary thinking that it is only removed with some difficulty. Everyone, of course, has his own intuitive perception of the duration of events and of their order in time, just as he has intuitive perception of bodies being extended in space and ordered or arranged in a certain manner. Earlier we saw that geometry and mechanics as deductive sciences are not concerned with the latter, but with conceptions such as point and line abstracted by the mind from them. So, also, are mechanics and theoretical physics concerned with "instants" and "intervals of time," the analogues of points and lines, which are not perceptual entities, but abstractions from our perceptions of duration and order in events. We also saw that while we could introduce any axioms and postulates we please into deductive geometry (provided they were consistent) we had to appeal to experiment to be satisfied that the conclusions drawn from them are true in nature. So from the point of view of pure theory we can make any consistent postulates we please about "instants," but, again, experiment must decide if the conclusions based on them are true in fact. The neglect to realise that it is not our own personal perceptions of events occupying a certain time and arranged in a certain order which are directly in question, but certain concepts drawn from these, causes a good deal of confusion in this connection. Our conscious perception of simultaneity of two events in our own time-order or the "before and after" arrangement in that order is in no way questioned or disturbed in any Relativity discussion. But just consider the nature of this perceived simultaneity. Two events which both "take time" more or less "overlap"; we really feel that a part of one and a part of the other are one and the same thing. In fact, an "event" is really the whole of Nature presented to our conscious perception at one moment, it is "here, now," and it is here not really instantaneously; it is present to our consciousness for some finite, if very short, time. We now begin to "analyse."

We think of the event as having " parts " in space. These " parts " are in the same very short time-stretch, so we say the " parts " are simultaneous, just as we might think of two bodies in space merging into one another, so that a part of one body was in the same place as part of another. That would be a " coincidence." Now, the latter idea we refine down to " coincident points," and the former we refine down to events occurring " at the same instant." It is these " events," with their properties of extension in time refined away, which are the subject matter of theoretical physics and its mathematical analysis.

Suppose, then, that we are engaged in " co-ordinating events "—*i.e.,* setting up a chart or table of time and place for all such events, we must make allowance for the gradual transmission of light. We see something happen now; does it happen really now? No, it happened a little while ago, or perhaps a great while ago; it all depends on how far away the place of the event was. If on the earth, extremely little; in the solar system, a matter of minutes; on a star, a matter of years, perhaps a very great number. Thus, the " physical time " of an event is not the time at which we perceive it, and for a satisfactory discussion of this feature of our observations it is no matter if it is a question of a millionth of a second before, or a million years before. The existence of this " time-lag " between event and perception has a considerable effect on the most convenient way of framing our physical laws, and on the nature of our measurements of time and space. Let the reader, therefore, get it well impressed on his mind. The time we are now going to talk about is " physical time " *inferred from* but not identical with time of perception. With this warning let us consider a simple occurrence. Denote a frame of reference and an observer by the letter A, and another frame of reference and observer by the letter B. A is fixed, and B is moving. " What!" you say, " isn't it just as true to say that B is fixed and A is moving?" Quite so, we shall consider B's case presently. What we mean is, that the distance between A and B or its direction (or both) is changing, and we are at the moment considering what is happening from A's point of view. A despatches a flash of light towards B, where there is a convenient mirror to reflect it back on the instant towards A. Now, as the reflection is assumed to occupy no finite time, the return journey takes the same time as the outward, for the speed of light is the same *in all directions*. So A regards the instant of arrival of the light at B as exactly midway between its instant of departure from and its instant of return to him. Now turn to B's point of view. Suppose, for the sake of a more definite picture, that A and B are getting further apart. B receives the flash of light and realises that as A is receding from him, A was nearer to him when the flash was despatched than he will be when it returns. So the return from B to A takes longer than the journey from A to B. Thus, B places the reception of the flash not midway between the instants of despatch and return but earlier than the mid-instant; for, remember, the velocity of light is the same *at all places*.

Now, what is impossible in this? A's perceptions are not directly apprehended by B, nor B's by A. No statement is made that two events which are simultaneous to A are also not simultaneous to him; although from the sort of remarks one hears bandied about in general conversation, many folk seem to entertain the quaint notion that somehow or other Relativity maintains such a preposterous proposition. The statements are not about A's perceptions at all, or B's, but about the inferred physical times in A's time-order and in B's time-order of the events which they perceive. The postulates have the entire support of physical experiment. The logic is incontrovertible. An escape from it can only be made by taking for granted that as B is going away from A, the light is aproaching him more slowly than if he were at rest. I have no doubt many of my readers will at first unconsciously make that step in the reasoning; but that step is fallacious; you must not make it. It contradicts *physical fact*. Another protest may emerge from outraged "common sense." "Oh, but surely clocks would show that this is wrong." What clocks? No clocks are sensitive enough to investigate such an imagined experiment on the earth. But please remember those parallel lines; how you thought it was quite sufficient to draw a couple of pencil strokes on paper, point your pencil "into space" and say: "Euclid must be true!" There is no use appealing to impracticable processes with clocks any more than impracticable processes with parallel lines; they tell us nothing. After all, physicists use "clocks" in their experiments, and very precise and sensitive they are, and what they find out can hardly be at variance with the actual behaviour of clocks, or how they might be supposed to behave in imagined if impracticable circumstances. It is, as a matter of fact, too restrictive on our ideas to attach our reasoning too much to the actual use of clocks and rulers. The beams of light are our ultimate clocks; that is the essence of Einstein's second postulate.

Let us try another exercise. Two people, A and B, stand on a road, and halfway between them is a source L, from which a flash of light can be sent out in all directions. They will admit that the instant at which a flash reaches A is simultaneous with the instant the flash reaches B. Two other people, X and Y, are standing on a long platform which is moving along the road, and X happens to be just at A when the flash reaches A, and Y at B when the flash reaches B. Consider the events from their point of view. L is not at rest in their "frame of reference," the platform. It is drawing nearer to one of them, say X, and receding from the other, Y. So when the flash started from L, it was not halfway between X and Y. So the light started from a place which, *when it started,* was further away from X than from Y. *It doesn't in the least matter what happened to L afterwards.* The initial conditions determine the result. The light requires a longer time to reach X than Y. So the arrival of the flash at X is later than the arrival at Y. Thus, two events simultaneous in the time-order of A and B on the road are not simultaneous in the time-order or people in another frame of reference moving with respect to the first frame.

Let us look at this still closer, assisting our ideas with a diagram or two.

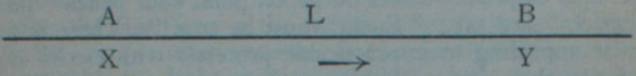

Fig. 1

Fig. 1 is a picture of the way things look to A and B when the flashes reach them. They might make chalk-marks on the platform. A would make his mark when X is opposite to him, and B when Y is opposite. They would be making these marks simultaneously; so they would say that the distance between the chalk-marks is equal to the distance between themselves. How would matters look to X and Y? X and Y do not admit that A came opposite X at the same time as B came opposite Y; A did not make the chalk-mark beside X when B made the chalk-mark beside Y. A made his mark later. Figs. 2, 3 and 4 show how X and Y view the occurrences

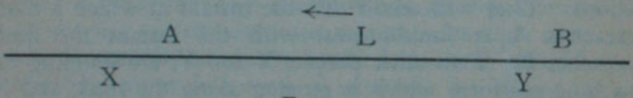

Fig. 2

when the light started from L.

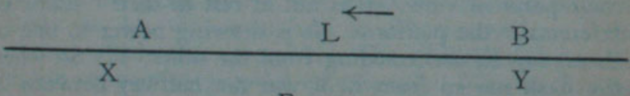

Fig. 3

when the light reached Y

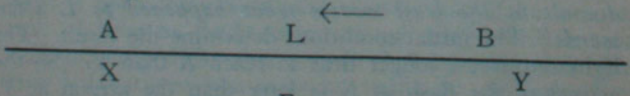

Fig. 4

when the light reached X

So X and Y say that the distance between A and B is shorter than the distance between the marks on their platform—that is, any material thing like the road which is in motion relative to X and Y is shorter for them than it is for people like A and B relative to whom it is at rest. Of course, if A and B choose to argue it out from their point of view, they say that the platform which is moving relative to them is shorter than it is for X and Y, to whom it is fixed. Every statement is capable of this reciprocal application. No one is privileged. They are both right.

This looks a bit like Fitzgerald contraction; but it really is not so. In that hypothesis the rod shortened by reason of its speed through the ether, its speed relative to any matter, had nothing to do with it. It was shorter, but it was *the same length to everybody*. On the relativity hypothesis the body has one length to A, another to X, still another to anyone who is travelling at another speed relative to A's frame in which the body is at rest. It is not really the correct way of looking at the matter to throw the blame, as it were, for finding these discrepancies in measurement on to rods and rulers. It is something inherent in our methods of observation and their relations to actual fact. Length and time are not absolute quantities.

Let us examine another instance. The source L on the road emits a flash, and a little later a second flash. The interval between the flashes is so adjusted that the departure of the second flash from the lamp is simultaneous with the arrival of the first flash at a place B on the road. The reader who is getting "on good terms" with all this will at once ask: "*To whom* are these events supposed to be simultaneous?" To the people on the road, A, B, and company, let us say. What about the people on the platform who are moving along the road in the direction L to B? To them the arrival of the first flash at B, and the departure of the second from L are not simultaneous events. The former event occurs earlier. The reader will realise this from what has just preceded, where he will notice that of two events which are simultaneous to

A and B, that one which happens at B occurs earlier for X and Y than the one which happens at A. These diagrams may help.

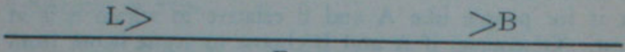

Fig. 5

As it appears to A, B, etc., when the first flash reaches B, and the second leaves L.

Fig. 6

As it appears to X, Y, etc., when the first flash reaches B.

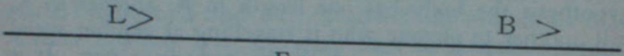

Fig. 7

As it appears to X, Y, etc., when the second flash starts from L.

Thus X and Y put the despatch of the second flash further on in time after the despatch of the first than do A and B. In short, X and Y find the interval of time between the despatch of the flashes from L longer than do A and B; but it should be noted that the two events happen *at the same place* for A and B, while this is not so for X and Y since L moves with respect to them between the events. In formal language this reads: two events occur at the same place in a certain frame of reference, but at different instants, and the interval between is measured by observers in this frame; these events are also observed by people in another frame, which is moving relative to the first; for the latter the events occur at different places, and the interval of time between them is longer. Of course, it is open to either party to throw the blame on the other fellow's clocks, saying that they are running too slow, but that betrays an unconscious leaning towards the older view that any clock can provide an absolute time.

We can now see why the old postulate about addition of velocities breaks down. A sees X moving in his (A's) frame of reference at ten miles per hour; X sees Z moving in his frame (X's) at five miles per hour in the same direction. But these estimates are in different space-time orders. You cannot just baldly transfer estimates made in X's space into A's space without modification. They can be "fitted together" no doubt, but not by simple addition. The result is not that Z is travelling at fifteen miles per hour in A's space; it is a little—a very little—less. The mathematics show that (using instead of 10 and 5, general symbols, u and v) the speed of Z in A's frame is not

$$u+v$$

but

$$\frac{u+v}{1+\frac{uv}{c^2}}$$

where c is the velocity of light. In ordinary cases, where u and v are very small compared to c, this amounts so nearly to $u+v$ as makes no practical difference; but if either u or v is comparable with c, there is a decided difference. Anyone with a little mathematics will see that if, for instance, $v=c$, then the result of the operation is just c, which is just what it should be; for light must travel at the same speed in A's frame as X's. When u and v have different directions the parallelogram rule does not hold; it must be modified in a manner which has a greater and greater effect on the result the greater u or v is.

This is entirely consistent; it would have been utterly opposed to our notions of continuity to have found ourselves in a position in which we had to assume the ordinary notions of composition of velocities for any speeds up to that of light and then a sudden breakdown in the rule at the speed of light. The rule breaks down from the very start, but only at high speeds does the discrepancy become practically important.

The reader will readily realise that this must involve a serious modification of the laws of dynamics. But we have already referred to the serious influence which the electromagnetic theory of matter has had on the postulate of the invariability of mass. We are rapidly approaching matters where it is impossible to proceed without mathematics involving the calculus, and we must content ourselves with the statement that Einstein, realising that, after all, Newton's laws of dynamics are an excellent approximation to the truth for slow speeds, suggested a very plausible generalisation of them, consistent with the Relativity postulates. One conclusion from them is that as a body increases in speed in a given frame, a given force produces in a given time an increase in velocity, which is always the same *as estimated in a frame in which the body is at rest at the moment;* but this means a smaller and smaller increase in the original given frame. In short, the mass increases with speed. The formula worked out agrees with the experiments on electrons referred to in the previous chapter, and is formally the same as Lorentz's. But Lorentz's speed is speed through the ether; the body has the same mass to everybody at one moment. Not so in Einstein's result. Its mass depends on its speed referred to the observer, and that may differ from one observer to another. Mass is no more an absolute quantity than distance or interval of time. Another result of Einstein's dynamics is that force is not an absolute quantity; the force on a body depends on the frame in which observations are made. He worked out the relations which must hold between the estimates of force in one frame and those in another having a given velocity relative to the first, and discovered that the forces involved in the electromagnetic theory of matter calculated from Maxwell's equations satisfy these relations. Thus, the whole region of dynamics (with its equations modified as he suggested) and electromagnetism showed itself consistent with the Relativity postulates. But gravitational forces, if governed by Newton's law, did not satisfy the necessary relations.

Before going on to deal with this difficulty, let us just glance at the very simple interpretation of the optical phenomena. Take aberration, for example. The star is moving in the earth's frame of reference—*i.e.*, in the earth's space. If you wish to observe it you point the telescope not to where it is now, but where it was when the light which you are now receiving left it. The formula deduced from this way of regarding the matter is just as satisfactory as any deduced from older views, and contains no symbol representing an unknown and unknowable velocity. Then, as regards the Döppler effect, we do not regard the earth as "breasting through the waves in the ether." On the stars periodic phenomena are occurring in the atoms. The spectroscope measures the periods for us. It is our "timepiece"; but it must yield a different value for the time of a given number of vibrations than it would if the source were at rest in our frame. So the corresponding lines in the star's spectrum are a little displaced from where they would be in the terrestrial spectra of the same element.

CHAPTER VII

THE GENERAL THEORY

EVERYONE is familiar with the mechanical experiences met with in carriages which are moving with increasing or decreasing speed along a road, or are sharply turning a bend, and most of us have seen what happens to people on the rapidly rotating "joy wheel." In common parlance, we say that we feel a force dragging us one way or the other in the carriage, or we say that a "centrifugal force" shoots the people off the wheel. To be sure, the dynamically educated point out that there are "really" no forces at all; what happens is the result of inertia, the tendency of the people to travel in the same line and at the same speed in space. And in the " space " of the road or hall that is true enough; but we have just realised that each moving thing has its own space—the carriage and the wheel, for example—and in those spaces the " forces " seem just as real as the ordinary force of gravitation. This result has an important effect on the Relativity hypothesis. These apparent forces—*i.e.*, apparent to people in the particular frame of reference—have a very decided resemblance to real gravitational forces; they produce the same acceleration in all unresisted bodies at the same place in the carriage or wheel. All the bodies free to move do so in the same direction, picking up speed at the same rate. This is obviously true from the fact that they are keeping their original speed relative to the road or hall.

Now, suppose that motion in an absolute space were really significant, what would be the result of an *acceleration* of the earth through space? (We have implicitly been restricting our considerations hitherto to uniform velocity.) In addition to the gravitational force due to the earth's matter and presumably determined by Newton's law of gravitation, we would experience further accelerative effects quite undistinguishable in their character from the acceleration of the real gravity. Still, we should be able theoretically to separate the total effect at any place into its two parts—the real Newtonian force and the apparent force; for Newton's law and a sufficient knowledge of the disposition and mass of the earth's material would enable us to calculate the former, and so infer from the total observed accelerations what the latter was, and thus gain some information about the acceleration of the earth's motion in space. It is not a question of the practicability of this process; such consideration as we have given to the matter will enable us to realise that we would have here a theoretical method for finding out something about absolute motion in space. It would suggest experiments to put it to the test.

Now, if such an experiment did succeed, it would clearly destroy the experimental basis of Relativity; for it would yield some information concerning absolute motion. Shortly after propounding his principle in 1905, Einstein realised this fact, and pointed out that if the Relativity principle is generally true, not only for uniform movement, but for any variable movement, then the reasoning leading to this supposed experimental conclusion must be wrong somewhere, and the only place where it could be wrong lay in the assumption that you could calculate the "real gravity" from Newton's law, or, indeed, from any law of an "action at a distance" where the force at a point is settled by its distances from the particles of the earth and their gravitational strengths. This daring suggestion he followed up by a search for a "Relativity law of gravitation," and after some years discovered one, completing his theoretical researches about 1915.

In this investigation he seized on one feature of the Newtonian law of gravitation, to which special attention was drawn in Chapter II.—the close relation between gravitational strength and inertial mass. With the intuition of genius, he saw that, although he might abandon Newtonian or approximately Newtonian formulæ involving masses and distances, he must cling to that close relation between gravitation and inertia. He realised that this was no mere accidental feature of gravitational forces; it was a distinguishing mark possessed by no force of different origin. At last in a flash of insight he leaped to the conclusion—gravitation and inertia are so closely related because they are aspects of the same natural fact. Gravitation is not the result of a "force," any more than inertia is. In absolute space, Newton postulated that there was only *one* path between two points possible for a body (*any* body), provided there was no force exerted on it. Well, we have no absolute space. Each small group of relatively fixed bodies has its own space. Furthermore, in a particular space there are a multitude of paths possible for a body between two positions (for example, on the earth you can project a body from one position so as to pass through another given position in an infinite number of ways), provided there are no mechanical forces such as pulls, pressures, resistances offered by solid surfaces or fluid media or electromagnetic forces exerted on it. But the reader will recall that in any space there is only *one* path between two positions which can be covered *in a given time* under those conditions—*i.e.*, one way of travelling for any body from one given position and given instant to another given position and given instant. Provided you link space and time in this way, the uniqueness of the path, characteristic in a more limited way of Newton's absolute space, remains true for any recognisable relative space. Of course, it is not now a matter of space alone; it involves space and time. But already we see from the special theory how the space and time of a given material frame is not to be identified with the space and time of another frame in movement with respect to the first. Again, the geometrical character of Newton's unique path was easily defined; it was straight. But we have also recognised earlier that this is not such a very definite criterion after all; it amounts to little more than saying over again that there is only one path, for it is a feature of straight lines that only one of them joins two given points. In any relative space the geometrical character of a gravitational path is not easily defined; but then, after all, the uniqueness of the occurrence does not lie in the path in space; it lies in the "*space-time*" path. We are not to think only of the points on the path; we must associate with each point a given instant. We are not merely to think of particles; we are to think of "event-particles," to use Whitehead's name—a particle at a given point at a given instant. Here we are beginning to enter that realm of ideas where the phrase "four-dimensional space-time" keeps recurring in discussion. Let us grasp its meaning.

In the space of a room we can conceive a great number of planes parallel to the floor drawn, each one with a definite number attached to it. Also a great number of vertical planes parallel to one pair of walls, also drawn and numbered, and a set parallel to the other pair of walls, treated likewise. Now, the position of any point in the room can be defined by the three numbers of the three planes on which it lies. (If it does not lie on a horizontal plane, say, but between two of them, draw more planes, and use decimals to extend your numbers). These are three "space-coordinates." Of course, it would only be practical convenience which would dictate these particular planes. The mathematician does not really limit himself this way; he can, if he likes, draw one set of parallel planes with any orientation, the second set also with any other orientation, and so on; or, indeed, for that matter, he need not use planes at all. He can use a "family" of any similar surfaces (spheres, for example, or cylinders, or cones) for determining a particular coordinate of any point. It is all one to him. He has a regular battery of mathematical analysis to deal with all this, invented for him by a few great geometers of the nineteenth century like Gauss and Riemann, and perfected for him by a succession of skilled mathematicians. It is really one of the beauties of mathematical analysis how the mathematical embodiment in symbolism of the broad geometrical truths of any space can be stated in a manner quite independent of the particular "mesh" of surfaces which one may use in any particular case to "co-ordinate" the space. So a space-path can be conceived as a succession of triplets of numbers, each triplet defining one point. Now attach to each triplet a fourth number defining the instant at which a particle was at the point. We then have a succession of quadruplets of numbers, each quadruplet defining an event-particle, the whole succession defining the space-time path.

But this is not the whole matter by any means. The special theory of Relativity enables us to entertain the idea that an interval of time can be "congruent" to a distance. We know that in geometry we imagine that lengths and figures can be moved about and superposed one on another to test if they fit in all parts—*i.e.*, are congruent. Now, the essence of the special theory is that there is a unique velocity; experimentally light has this velocity, or, at all events, one so near it that practically we take the velocity of light to be this unique velocity.* So in a special sense a distance between two points can be said to be congruent to an interval of time if the distance has the same length as that covered by a body, travelling with this unique speed, in that interval. Now, in what follows we shall suppose that when we speak of a "time-coordinate," this process is in mind. The number representing the time in question—*i.e.*, between some event and a definitely assigned "time-origin" or starting instant—has been translated through multiplication by the number representing the unique speed in suitable units into a distance. We now have the four coordinates of an event particle in space-time, and our mathematics works out as if we were actually manipulating the analysis of position and separation in a four-dimensional space. That is what we mean by "four-dimensional analysis." The reader is not asked to do the impossible—see or feel four dimensions in the way he sees or feels three.

It was not really Einstein himself who was the first to realise just how beautifully his special Relativity lent itself to this concept of space-time. That was the work of the mathematician Minkowski, who delivered a famous address on the matter to a group of scientists at Cologne in 1908. But, once having realised it, Einstein made powerful use of the idea in his far-reaching suggestions concerning gravitation.

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* In the special theory, Einstein's laws of dynamics show that the mass of any body would become infinite in a frame if it attained this unique velocity in that frame—*i.e.*, its velocity cannot exceed this velocity, and cannot really in practice attain it, although it might come very near to it. It is conceivable, therefore, that light may not just reach the ideal limit, but experiment certainly gives us no clue, as yet, to the very minute difference, if any such difference really exists.

Minkowski had laid special emphasis on a most important result. In each frame of reference there is a certain time-lapse between two events, and a certain distance between the places where they occur; there is nothing special about these measures for those two definite events. They have different values in another frame in motion with respect to the former. But there is a constant "separation" of the events in space-time, no matter in what special frame you observe them; for if you subtract the square of the distance between the events from the square of the "time-stretch" (*i.e.*, the interval translated in the manner just indicated), the result is the same, no matter what frame you choose; that is a result easily deduced from the relations of time and space between two frames which were, in a sense, first discovered by Lorentz, but correctly interpreted by Einstein. The square root of this difference was called by Minkowski the "proper time" (*Eigenzeit*) between the events, but the words "separation," "absolute interval," are more commonly used. Out of the ruin of absolute ideas—length, time, mass, force—there emerges this "absolute"; for it is the same for all observers. The mathematical process involved in calculating the separation from the time-coordinates and space-coordinates bears some resemblance to the method of calculating the hypotenuse of a right-angled triangle from its sides, except that we have a minus instead of a plus sign, and we recognise the latter process at work in calculating the distance separating two points in a room by taking the "component" of the distance parallel to one side of the room, the component parallel to the other, and the component vertically upwards. As a matter of fact, the minus sign is of no importance *in the general mathematical analysis*. Those who know something of "imaginary quantities" in mathematics will possibly realise this. (The reader must not misunderstand this statement; the minus sign is of enormous importance in *physical fact*. It symbolises the essential difference between our time-perceptions and our space-perceptions.)

Here was the idea that Einstein seized on. In a given frame there is a unique space-time path for any body between this position and instant and that position and instant. We idealise the body as a particle. The points and instants are the constituents of two event-particles. The uniqueness of the path is settled by some property of their separation in space-time, just as in Newton's absolute space the uniqueness of the space-path was settled by its straightness. What property?

Here in the final stages of our attempt to "understand Einstein" we come up against that famous phrase "curved or twisted space." Unfortunately most of the newspapers got it quite wrong. It should be "curved space-time"—a quite different idea. No wonder the man-in-the-street thought that he was meeting a new St. Paul. "Behold, I tell you a mystery!" But nothing could have been further from Einstein's mind than mysticism. Here was the general run of his ideas:

On a plane surface there is one line between two points easily distinguished from all others; it is the straight line. But on *any* surface you can also mark out an easily distinguished line between two points, the line of shortest length; it is, in general, curved, of course, but if you measure the various joining lines, you can pick out the shortest. Notice how you have to measure. Each joining line is really regarded as a series of short, straight lines, like part of an inscribed or exscribed polygon, and each little bit is measured and the lot added. That is really what is done to such degree of fineness in division as is practicably necessary. Give the mathematician sufficient information about the surface, and he will show you how to do it theoretically to the utmost degree of fineness conceivable. He does that, for instance, when he shows you how he could work out the famous ratio of the circumference of a circle to its diameter to hundreds of places of decimals if it were worth anyone's while. So there is a recognisable uniqueness about the line of shortest length between two points. There is also a similar recognisable property about the line of longest length on some surfaces. Thus, Liverpool and Edinburgh are situated nearly on the same meridian of latitude. We recognise that the shortest way from Liverpool to Edinburgh is to travel due north; while if we travelled due south to the South Pole, and then continued "straight on" to the North Pole, and then "straight on" to Edinburgh, we should take the "longest way round." These recognisable lines of least and greatest length on a surface are called "geodetics." The mathematician again comes to our rescue. He has a very general theory of the way to deal with any surface by means of mathematical symbolism, so as to express in this symbolism the distance between two points along any line, and thus to select the geodetics from among all the lines. Incidentally he conceives a set of non-intersecting curves drawn on the surface and numbered, and then another set of curves crossing these and also numbered (just like lines of latitude and longitude on a globe). These curves enable him to coordinate his surface. Each point has two coordinates, the numbers of the two curves on which it lies. The theory of his analysis does not depend on what curves he draws. (The convenience of the application of his theory to actual cases, of course, does.)

This analysis is, of course, connected with *two* dimensions; two coordinates define each point. Now, what Einstein suggested was that the same type of analysis should be applied to events in space-time to select the gravitational "tracks" between one event-particle and another. Most readers will probably have solved simple simultaneous equations at school and remember how, when they had learnt the way of "doing the sums" for x and y, two "variables," they quite easily recognised how to do it for x, y, z, three variables, or for four variables, x, y, z, u, and so on. Unfortunately, "Tensor Analysis" is not couched in the simple symbolism of elementary algebra, and so it is hopeless to give the general reader any idea of its method and scope; but it is equally true that, once having mastered its methods for two variables—the co-ordinates on a surface, for example

—it is entirely a matter of the most natural development to extend it to deal with three variables (coordinates in space, for example), or, more germane to our purpose, four variables (the coordinates of event-particles in space-time, for example).

Here, then, was the mathematical method; but that was not enough. The mathematician can make no practical application of this analysis which can be tested by actual measurement, unless you tell him what surface or what type of surface you want it applied to. Now, what Einstein recognised was that all the mathematics of his earlier, special theory was, when dressed out in this four-dimensional Tensor analysis, just like the analysis with two variables of the geometry of a *plane* surface. In his earlier theory, gravitation was implicitly ruled out and not considered, because it was supposed to be due to a force which satisfied a certain kind of formula which could not fit into the scheme. Well, what more natural than to assume that, in order to bring in gravitation, you had to use the four-dimensional analysis in the rather more complex form analogous to the analysis employing two variables when used for treating *curved* surfaces. This brilliant notion turned out quite correct, and is the origin of the famous phrase "curved space-time."

But what about this "law of gravitation"? Dear reader, it can't be done. How could it be? You don't know, I presume, the first word about Tensor analysis; how can anyone write down an expression in its symbolism of whose meaning you would have the slightest idea? All one can say is this: "You are well aware that all planes are like one another, that all spheres are like one another, all cones, all cylinders, all egg-shapes, all ring-shapes, all saddle-shapes." The mathematician has a way of dealing with all shapes, because he has a way in his analysis of defining the special features of a particular shape. It is closely connected with what is called "the measure of curvature." So, on à priori grounds, all sorts of properties defined in analogous ways in four-dimensional Tensor analysis might be postulated of space-time;

but experiment would have to settle which were true in fact. Einstein made a plausible guess. Far away from matter, space-time is "flat"—*i.e.*, the actual events go on as the mathematics of the special Relativity would work them out. Coming nearer matter, the "curvature" begins to manifest itself in a particular way, getting more and more pronounced till we reach the matter. Einstein suggested an exact rule connecting the "curvature" at a certain place *and time*, and the material content of the place at that time. The "curvature" is, of course, a convenient term for a group of mathematical expressions analogous to those similar expressions which in the geometry of surfaces do actually represent curvature. Figuratively speaking, Einstein's law of gravitation postulated something about the behaviour of the curvature of our world space-time.

The law was capable of being tested in three ways. Firstly, the path of a *single* planet round the sun should not be the ideal ellipse round the sun, but differ from it in a calculable way. There are certain facts about the motion of the planet Mercury which offer considerable support to this result. Secondly, a ray of light passing near to a large mass like the sun should be deviated from straightness in the earth's frame of reference by a calculable amount. This famous experiment is probably familiar to the reader. The results acquired during the last two total eclipses of the sun are entirely confirmatory. Thirdly, the spectral lines of a star should suffer a displacement towards the red end of the spectrum (quite distinct from the displacement due to relative motion, which may be towards the violet or red—the Döppler effect), which is calculable if the mass and distance of the star are known. This has proved the most difficult test. It is now, however, regarded as a very satisfactory verification of Einstein's ideas.

One word in conclusion. We began this chapter by pointing out how gravitative effects could be simulated by occupying an accelerated frame of reference. A man drops a stone from a carriage. It travels in a straight line from him. It travels in a parabola to a man on the footpath. But these are only different aspects of the same *space-time track*. The mathematician tells you that the men are like two people, each with a model in some textile material of the same surface. One of them now changes the shape of his model without tearing, straining, or crumpling the material, just the way you would fold, for example, a plane sheet into a cylinder or cone. It has a different " look," but its essentials are the same as before. Two marks on it, for instance, are at the same distance apart. The mathematical measure of its " curvature " is the same as before. " The more it changes, the more it is the same thing." That is true of space-time. Because we are essentially beings whose perceptions can only grasp events " at one moment," and we can only think of all events in a time-order, we can therefore only obtain glimpses of the underlying reality, which are entirely determined by our own individual relation to this reality.

" And what has all this got to do with me and my affairs? What change is it going to make?"

None at all in the sense in which certain other famous discoveries have made immediate and obvious changes. The discovery of electromagnetic induction brought the dynamo and motor in a few years. Maxwell and Hertz were the real founders of broadcasting. X-rays have changed the whole course of medical and surgical technique. But nowhere in the whole range of scientific applications to the material and economic affairs of life can one see a single place where the satisfactory removal of those minute discrepancies between theory and experiment, which worried the physicists for a generation, can have the slightest effect.

But " man does not live by bread alone." We now recognise that in the gradual destruction of the medieval faith that man was the chosen creation of God, at the centre of the universe, all Nature assisting towards the accomplishment of his ends, no influence was more potent than the rise of the Newtonian philosophy. There is a grim irony in the fact that the same man whose piety and theological learning were so marked a feature of his later life should have so signally contributed towards the feeling that, if our earth is a little tiny planet in this infinite space through which roll those mighty spheres in eternal cycles, then we must be small fry indeed. What possible effect can our little lives and experiences have on this mighty drama? We are but the "product of causes which had no prevision of the end they were achieving." Nature goes on its way regardless and unaffected by our lives with their hopes and fears and beliefs.

Now, it appears *we* are at the centre of *our* universe in a very special and individual fashion. We began our scientific life as a race with the belief in the absolute fixity of the earth, then of the sun; then Newton swept all that away; nothing was fixed. Now we don't mind; it doesn't matter. We can choose anything we like as our "fixture." Our earth is as good as any other body—and better, for it is more convenient for us in many ways. Indeed, each individual is his own "fixture," but fortunately our slow relative speeds to each other make the practical adjustment of our separate spaces and times a matter of no trouble, and so we naturally choose the earth's space as fixture. We have all talked about whirling "through space" at so many miles per second, but that has seemed to have had pretty little effect on our lives, although it has had, cumulatively through generations, a serious effect on our attitude towards life.

Who can tell what this change in our cosmological views will have on our attitude to the problems of human life and destiny? We are all aware of the revolt in philosophy against the materialism of the nineteenth century. Gradually the biological and social sciences have followed the philosophers in their search for a view of life which will recognise that man's life and activities are not the mere by-product of natural forces which go on blindly, regardless of insignificant midgets on one of the tiniest members of the immense stellar concourse. Only physical science seemed to stand out against this humanistic movement. Not so, any longer. How can it be otherwise when the aspect of every natural event is as closely related to the particular individual observing it as Relativity shows?

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