THE MYSTERY OF SPACE

The domain of the senses, in nature, is almost infinitely small in comparison with the vast region accessible to thought which lies beyond them.—TYNDALL, "On Radiant Heat."

THE MYSTERY OF SPACE

A Study of the Hyperspace Movement in the Light of the Evolution of New Psychic Faculties

and

An Inquiry into the Genesis and Essential Nature of Space

BY

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TO
THE CHERISHED MEMORY
OF
Mylie De Prè
WHOSE WIFELY DEVOTION,
SYMPATHETIC ASSISTANCE AND ENCOURAGEMENT
DURING THE EARLY LABORS ON THE TEXT
WERE A CONSTANT SOURCE OF
INSPIRATION AND FORTITUDE TO THE AUTHOR,
THIS BOOK IS DEDICATED
AFFECTIONATELY

PREFACE

MATHEMATICS is the biometer of intellectual evolution. Hence, the determination of the *status quo* of the intellect at any time can be accomplished most satisfactorily by applying to it the rigorous measure of the mathematical method. The intellect has but one true divining rod and that is mathematics. By day and by night it points the way unerringly, so long as it leads through materiality; but, falteringly, blindly, fatally, when that way veers into the territory of vitality and spirituality.

Wherefore, when we have wished to ascertain the real status of the intellect, as well as its limitations, tendencies, possibilities, we have turned to its conduct in the field of mathesis where it is least trammeled in its ingressive and egressive motivations because of the natural and easy accommodation which is offered for intellectual movement. Whether there are signs of moribundity or symptomatic evidences of marked growth or of a termination of intellectual regnancy, or whatever may be the occasion for the examination, no surer index than the mathematical may be found for the purpose. Full logical justification is, therefore, claimed for the choice of mathematical evidences to test the assumption that a new era of conscious mental

conquest is opening for the vanguard of humanity and sequently for the human family.

The treatment of these evidences has fallen logically into two divisions, namely, the first, a brief and elementary review of the principles of the non-Euclidean geometry and their bearings upon the question of space as the subject of mathematical study; and second, the consideration of space as a psychological, vital and dynamic or creative phenomenon.

In Part I an effort has been made to trace the growth of the notion of hyperspace and to show that it is a symbol of a new epoch of intellectual expansion, an actual seizure of a new domain of awareness by the mind. And for this purpose a critical examination of the fundamental question of dimensionality is entered upon from which it develops that the status of this primary norm of mathematical thought exhibits a relatively inchoate character because of its insufficiency as a definitive quantity, and further, because of its rather superfoetated aspect when utilized as a panacea for mathetic symptoms. Also, it has been found necessary to survey the field of the four-space which has been accorded such a prominent place in the mathematical thought of the day. The reader should find in the chapter devoted to it adequate material for thought and sufficient comprehension of its meanings as a mathematical contrivance.

In Part II an endeavor is made to interpret the evidences offered by high mathematic considerations in the light of the larger psychogenetic movement. For this undertaking the quality of awareness has been studied with the view to establishing its priority as a determining agency in the consideration of space in

aspects broader and less restricted than those embraced in the mathematical premises. Wherefore, it appears that there are massive implications arising out of the hyperspace propaganda which have hitherto been neglected in the preliminaries incident to the fabrication of its structure. A very brief, and more or less symbolic, sketch of the genesis of space has served to demonstrate its essential nature as a consubstantive of materiality, vitality and intellectuality, the three major objective processes. Consequently it becomes imperatively necessary that any view of space which neglects its kosmic intent and purpose should be regarded as gravely fragmentary and insufficient. It is only by linking up the two aspects of space, the mathematical and the psychological, in such a manner that the one shall supplement the other, that we shall be able to arrive at a truly satisfactory understanding of its nature.

In Chapter IX attention is invited to some of the extremities of mathematical laws wherein it is shown that, because mathematical goods are strictly of intellectual texture and fabric, vain is the hope of reaching any reliable certainty with respect to many vital questions, even regarding space itself, by means of the mathematical method. The intellect, and, therefore, mathematics encounter the most formidable stricture when effort is made to maneuver in the field of vitality or realism. In addition, it is shown that, when pushed to the utmost logical limits, metageometry proves not only futile, but emphasizes the need for a sharp turning of the path of search from the intellectual or material to the spiritual or intuitional. Indeed, it becomes painfully certain that the Golden

Fleece of profounder knowledge will be discovered never by an expedition whose bark has its sails set for the winds of mathematical seas. But, contrarily, a new bark, moored at the furthermost shore of the sea of intellectuality with sails set for the winds which come from the realm of intuitional perception, must be seized. Whereupon, by the straightest line, we shall, at the last, land upon the shore of realism, of truth all inclusive.

Mathematical evidences have been used in these discussions because they, of all lines of knowledge, afford a more just exemplification of intellectual evolution. The science of mathematics is the measure of the quality of intellectual growth and, therefore, its data, its postulates, hypotheses and advances clearly mark the stages of the intellectual movement.

Chapter X is the natural and logical sequence of the inquiry into the question of spatiality. The conclusions reached therein and the obvious inferences which should be drawn from the arguments presented flow inevitably not only from the evidences of mathematical data but of the common observations of life. And while we disclaim any intention of demanding acceptance of them as final, authoritative declarations, we shall be satisfied if the readers of this volume be incited to solve for themselves the problems which these queries naturally suggest. Happy indeed shall be the outcome if there be any who, following the path sketched herein, shall find the solution of the *Mystery of Space* and apply its meanings to the enhancement of the values of the intuitive life.

In conclusion, the author esteems it a special obligation of gratitude that he should here acknowledge

the debt which he owes to all of his friends who have in any way assisted or encouraged him in the completion of this work. Among those whom he is permitted to thank in this way is Mr. JAMES RINDFLEISCH who, having very kindly prepared the illustrations for the photo-engravers, is deserving of special mention.

ROBERT T. BROWNE.

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CONTENTS

PAGE

PREFACE vii

INTRODUCTION: Explanatory Notes..... 1

PART ONE

CHAPTER I

THE PROLOGUE

On the Variability of Psychic Powers—The Discovery of the Fourth Dimension Marks a Distinct Stage in Psychogenesis — The non-Methodical Character of Discoveries—The Three Periods of Psychogenetic Development—The Scope and Permissibility of Mathetic License—Kosmic Unitariness Underlying Diversity 23

CHAPTER II

HISTORICAL SKETCH OF THE HYPERSPACE MOVEMENT

Egypt the Birthplace of Geometry—Precursors: Nasir-Eddin, Christoph Clavius, Saccheri, Lambert, La Grange, Kant—Influence of the *Mecanique Analytique*—The Parallel Postulate the Root and Substance of the Non-Euclidean Geometry—The Three Great Periods: The Formative, Determinative and Elaborative—Riemann and the Properties of Analytic Spaces 44

CHAPTER III

Essentials of the Non-Euclidean Geometry

PAGE

The Non-Euclidean Geometry Concerned with Conceptual Space Entirely—Outcome of Failures at Solving the Parallel-Postulate—The Basis of the Non-Euclidean Geometry—Space Curvature and Manifoldness—Some Elements of the Non-Euclidean Geometry—Certainty, Necessity and Universality as Bulwarks of Geometry—Some Consequences of Efforts at Solving the Parallel-Postulate—The Final Issue of the Non-Euclidean Geometry—Extended Consciousness 69

CHAPTER IV

Dimensionality

Arbitrary Character of Dimensionality—Various Definitions of Dimension—Real Space and Geometric Space Differentiated—The Finity of Space—Difference Between the Purely Formal and the Actual—Space as Dynamic Appearance—The *A Priori* and the *A Posteriori* as Defined by Paul Carus..... 92

CHAPTER V

The Fourth Dimension

The Ideal and the Representative Nature of Objects in the Sensible World—The Fluxional, the Basis of Mental Differences—Natural Symbols and Artificial Symbols—Use of Analogies to Prove the Existence of a Fourth Dimension—The Generation of a Hypercube or Tesseract—Possibilities in the World of the Fourth Dimension—Some Logical Difficulties Inhering in the Four-Space Conception—The Fallacy of the Plane-Rotation Hypothesis—C. H. Hinton and Major Ellis on the Fourth Dimension..... 118

PART TWO

SPATIALITY: AN INQUIRY INTO THE ESSENTIAL NATURE OF SPACE AS DISTINGUISHED FROM THE MATHEMATICAL INTERPRETATION

CHAPTER VI

Consciousness the Norm of Space Determinations

PAGE

Realism Is Determined by Awareness—Succession of Degrees of Realism—Sufficiency of Tridimensionality—The Insufficiency of Self-consistency as a Norm of Truth—General Forward Movement in the Evolution of Consciousness Implied in the Hyperspace Concept—The Hypothetical Nature of Our Knowledge—Hyperspace the Symbol of a More Extensive Realm of Awareness—Variations in the Methods of Interpreting Intellectual Notions—The Tuitional and the Intuitional Faculties—The Illusionary Character of the Phenomenal—Consciousness and the Degrees of Realism 161

CHAPTER VII

The Genesis and Nature of Space

Symbology of Mathematical Knowledge—Manifestation and Non-manifestation Defined—The Pyknon and Pyknosis—The Kosmic Engenderment of Space—On the Consubstantiality of Spatiality, Intellectuality, Materiality, Vitality and Kosmic Geometrism—Chaos-Theos-Kosmos — Chaogeny and Chaomorphogeny—N. Malebranche on God and the World—The Space-Mind—Space and Mind Are One—The Kosmic Pentoglyph 203

CHAPTER VIII

The Mystery of Space

PAGE

The Thinker and the Ego—Increscent Automatism of the Intellect—The Egopsyche and the Omnipsyche—Kosmic Order or Geometrism—Life as Engendering Element—The Mystery of Space Stated—Kathekos and Kathekotic Consciousness—Function of the Ideal—The Path of Search for an Understanding of the Nature and Extent of Space Must Proceed in an Inverse Direction 242

CHAPTER IX

Metageometrical Near-truths

Realism Is Psychological and Vital—The Impermanence of Facts—On the Tendency of the Intellect to Fragmentate—The Intellect and Logic—The Passage of Space—Kosmometer and Zoometer, Instruments for the Measurement of the Passage of Space and the Flow of Life—The Disposal of Life and the Power to Create—Space, a Dynamic, Creative Process—Numbers and Kosmogony—Kosmic Significance of the Circle and the Pi-proportion—Mechanical Tendence of the Intellect and Its Inaptitude for the Understanding of Life—The Criterion of Truth..... 284

CHAPTER X

Media of New Perceptive Faculties

The Spiritualization of Matter Is the End of Evolution—Sequence and Design in the Evolution of Human Faculties—The Upspringing Intuition—Evidences of Supernormal Powers of Perception and the Possibility of Attainment—The Influence and Place of the Pituitary Body and the Pineal Gland in the Evolution of Additional Faculties—The Skeptical Attitude of Empirical Science and the Need for a More Liberal Posture—The General Results of Pituitarial Awakening Upon Man and the Theory of Knowledge..... 327

Bibliography 359

Index 367

THE MYSTERY OF SPACE

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INTRODUCTION

Explanatory Notes

The following interpretation of words, phrases and notions occurring in the text, and also biographical sketches which the scope and purpose of the book itself make it impracticable to elaborate, are appended with the view to facilitating its perusal.

AT-ONE-MENT (state of unity, unitariness); denotes the ultimate state of oneness towards which all evolutionary movement tends; applied to consciousness, indicates the final expansion of consciousness wherein it coincides with the universal consciousness in extent and quality of comprehension. As applied to things, denotes the unification of all movements, tendencies, and evolutions as a singularity; the end of all evolutionary activity (vide p. 270).

BELTRAMI, EUGENIO, was born at Cremona, November 16, 1835; there he attended the elementary schools, the gymnasium and the lyceum, excepting the scholastic year 1848-49 when he was at the Gymnasium of Venice, now known as Marco Polo. He finished his lyceal studies in the summer of 1853, and in the following autumn (November) became a student in the *Mathematical Faculty of the University of Pavia,* after having obtained a scholarship there on the Castiglioni Foundation in the *Collegio Ghisleri.*

In 1854, the succeeding year, he was expelled from this college in company with five of his colleagues who were accused of promoting "disorders" against the Abbot LEONARDI, rector of the college. The expulsion brought him many hardships and disappointments, and for two years he drifted along merely existing as his family was too poor to have him matriculated at another university. But in 1856, he went to Verona where he succeeded in securing employment as secretary to the engineer, DIDAY, in the Government service of Lombardy-Venice. On January 10, 1857, he was dismissed from this position "for political reasons"; but as the annexation of Lombardy to Piedmont occurred soon thereafter, he became again attached to the office of DIDAY, his former employer, when it was transferred to Milan as a consequence of political changes.

At Milan BELTRAMI took up his mathematical education in real earnest as he now had access to Professor BRIOSCHI, his former tutor, and also LUIGI CREMONA. Through the influence of these two men he was designated (October 18, 1862) "Professore straordinario" in the University of Bologna. His work on *Surfaces of Constant Negative Curvature,* as the pseudosphere, and his application of the expression given by LOBACHEVSKI (q.v.) for the angle of parallelism, very definitely secure for him a place among the foremost workers in the field of the non-Euclidean geometry. He postulated a theorem, known as *Beltrami's Theorem,* which he stated as follows: "The center of a circle circumscribing a triangle is the center of gravity of the centers of its inscribed and escribed circles." He died in the year 1900. (Vide *Amer. Math. Mo.,* Vol. IX, p. 59.)

BOLYAI, JANOS (1802-1860), was born at Kleansenburg, Hungary. He is said to have inherited his mathematical genius from his father, BOLYAI FARKAS (1775-1856), who was born at Bolya, Hungary. Being a very spirited youth, his progress in his studies was most remarkable. He completed the curriculum at the Latin school when only twelve years of

age. Was graduated from the Philosophical Curriculum as a result of two years of study and then entered the Viennese Academy of Engineers. Was appointed lieutenant at Temesvárlin, 1823, whence on November 3, 1823, he wrote his father: "I have discovered such magnificent things that I am myself astonished at them. It would be damage eternal if they were lost. When you see them, my father, you yourself will acknowledge it. Now, I cannot say more, only so much: that from nothing I have created another wholly new world." This letter was written in the Magyar language and has been preserved at the Marcos Vásárhely, Hungary. The mathematical conceptions formulated by him became the appendix of the *Tentamen,* a book which his father had written on the Theory of Parallels.

His *Science Absolute of Space* was translated into the French in 1868 by the French mathematician, J. HOÜEL, to whom belongs the credit of popularizing the works both of BOLYAI and LOBACHEVSKI. (Vide *Science,* n. s., Vol. 35, No. 906, 1912.)

CAYLEY, ARTHUR, born at Richmond, Surrey, England, August 16, 1821; studied at King's College school; entered Trinity College, Cambridge, already a well equipped mathematician at the age of seventeen. When but twenty-one years of age he took two of the highest honors in the University of Cambridge. He was Senior Wrangler and First Smith's Prizeman. He published his first paper in 1841 and this was followed by eight hundred memoirs.

For fourteen years he practiced as Conveyancer. In 1863 Lady SADLER'S various trusts were consolidated, and a new Sadlerian professorship of Pure Mathematics was created for the express purpose of affording a place for Cayley. Meanwhile, as early as 1852 he was a fellow of the Royal Society; in 1858 he joined SYLVESTER and STOKES in publishing the *Quarterly Journal of Pure and Applied Mathematics.*

He was for a considerable time principal adviser as to the

merits of all mathematical papers which were presented for publication to the Royal Society, the Astronomical Society, the Mathematical Society and the Cambridge Philosophical Society. He is said to have been the "most learned and erudite of mathematicians," and much of the material, therefore, which now constitutes the basis of the non-Euclidean geometry is due to his laborious efforts and comprehensive knowledge of mathematics. (Vide *Review of Reviews,* Vol. II, 1895, Sketch, reprinted from *Monist.*)

CHAOGENY (Gr. Chaos, disorder, geny—generating, evolution); the evolution of chaos into order. A kosmic process involving the elaboration of the original, formless world-plasm into the first faint signs of orderliness; the beginnings of the movement of life or the Creative Logos in preparation of the field of evolution.

CHAOMORPHOGENY (Gr. Chaos, disorder—Morphe, form—geny, becoming, generating); evolution of the space-form, the universe; the establishment of the metes and bounds of the universe; also, the origination and characterization of all forms as to tendence, purpose and limitations.

CONCEPTUALIZATION—The act of conceptualizing, the formulation of concepts; the process by which the Thinker arrives at concepts; the logical procedure by which the consistency of a scheme of thought is established.

CONSTRUCTION, IDEAL—A purely formal conception; a theory, hypothesis; a logical determination not necessarily based upon facts, but possessing virtue because of consistency; a self-consistent scheme of thought.

COSMOS—Whenever the term "cosmos" appears in the text spelled as here shown, it refers to phenomena pertaining to the earth or the solar system; when spelled "kosmos" reference is made to the universe as a whole.

CRITERION OF TRUTH—Defined in the text as a four-fold standard of reference, embracing the following elements, namely, the causal, the sustentative, relational and developmental. Lacking any one of these, no view of truth is more than fragmentary. Applied to space, it contemplates an inquiry into the genesis or causal aspect, an accounting for the duration aspect, a recognition of its relation to the totality of objects, and lastly, a prophecy of its telestic or perfective culmination. This test has been applied to the study of space as sketched in the text and the conclusions reached are an outcome of the inquiry directed along these lines.

CURVATURE OF SPACE—A doctrine formulated by RIEMANN and which maintains that space is curved, and consequently, all lines drawn therein are curved lines. Professor PICKERING aptly describes the results of movements in a curved space by pointing out that if we go far enough east we arrive at the west; north, we arrive at the south; towards the zenith, we arrive at the nadir, and *vice versa.*

DEIFORM—The basic idea indicated is that the universe is the form or body of the supreme deity, since He is not only immanent in the kosmos, but sustains it by His life; that in order to create a manifested universe, it was necessary to limit, or sacrifice, in a measure, His own illimitability. Viewed in this light, the kosmos assumes an added significance.

DIMENSION—(L. Dimensio, to measure), measurement; a system of space measurement. The Euclidean geometry recognizes three dimensions or coördinates as being necessary to establish a point position; witness, the corners of a cube to form which three of the edges come together at a point. These edges represent coördinates. For the purposes of metageometry, the term dimension has been variously defined, as, direction, extent, a system of space measurement, or a system of coördinates. Regarded as a series of coördinates, it became possible to postulate a system which required four coördinates to estab-

lish the position of a point, as in the hypercube. There may be five, six, seven, eight, or any number of such coördinate systems according to the kind of space involved in the calculations. Determinations based upon the logical necessities of the various coördinate systems have been found to be self-consistent throughout and, therefore, valid for metageometrical purposes. Much depends upon the definition; for, after the definition has been once determined it remains then merely to make inferences and conclusions conform to the intent of the definition.

DIVERSITY—Philosophically, the idea indicated has reference to all dissimilarities, differences, inequalities, divergent tendencies, movements and characteristics to be noted in the universum of life; the antithesis of kosmic unity; the natural outcome of life in seeking expression; the result of the fragmentative tendency of life.

DUODIM (duo, two; dim, abbreviation of dimension)—A hypothetical being supposed to be possessed with a consciousness adapted only to two dimensions; a dweller in "Flatland" or two-space whose scope of motility is limited to two directions, as on the face of a plane; a term invented by hyperspace advocates for the purpose of establishing by analogy some of the characteristics of the four space and also its rationality.

DUOPYKNON (duo, two—pyknon, primary unit in the process of kosmic involution, a condensation)—Secondary phase in the elaboration of chaos into kosmic order. DUOPYKNOSIS (duo, two, secondary—pyknosis, process of condensation and origination)—The second period of the involutionary movement of life during which the duadic plane of the kosmos is being established; the second, in the series of seven distinct phases, of space-genesis; dual differentiation of kosmic plasm. Duopyknosis contemplates that, in the passage of the kosmos-to-be from the plane of non-manifestation to the plane of manifestation, there are seven distinct, though interdependent and

interrelational, stages through which life passes, and that, of these, it is the second. It relates to the plane of non-manifestation and is, therefore, beyond the ken of the intellectuality, being a symbol.

EGOPSYCHE (Ego, the self-conscious I—psyche, soul)—The mental, emotional and physical mechanisms of man, the Thinker. These include the purely mental system, the emotional or affective mechanism, the nerve-systems (cerebro-spinal and sympathetic) and the brain; the objective or sense-derived consciousness of the Thinker which is elaborated from the total mass of perceptions transmitted through the senses; the medium of self-consciousness; the intellectual consciousness as distinguished from the intuitional or omnipsychic consciousness (q.v.).

The notion of the egopsychic consciousness is based upon data already empirically determined from the mass of evidences everywhere observable. It seems to be apparent that there is a consciousness, a seat of knowledge, in man the content of which is unknown to the sense-consciousness. Dreams, premonitions, intuitions, impressions and the totality of all such phenomena substantiate this view. Furthermore, it is agreed that the source of the intuition is not identical with that of the intellect. The egopsychic consciousness, accordingly, is purely intellectual.

FLUXION, PSYCHIC—The difference between a mental image and an object; an image is the representation of certain salient or cardinal characteristics of an object, sufficient for identification; but an image is not congruent, in every respect, with the object. Thus when we perceive an object, although as BERGSON contends we perceive it in the place where it is and not in the brain, it is the image of the object which takes its place in memory and not the object itself. There is, of course, a marked disparity between this memory-image and the object. Even if the image possessed one of the properties

of the object, as, size, it could not take its place in memory, and neither could it do so if it possessed any of the properties of the real object. Consciousness is such that all due allowances are made for these conditions and the mind is able to retain more or less exact knowledge of these properties in the image; but there is a difference, small though it may be. This difference is the psychic fluxional.

FOHAT (Skt)—A term applied to the Creative Logos who is said to be the generating element in the differentiation of chaos into kosmic orderliness; the supreme deity in the rôle of Creator.

FORM, PURE—An abstraction arrived at by subtracting the last vestige of materiality or substantiality from an idea and viewing the remains as a pure unsubstantial form or idealization; the shell or frame-work of a material object or condition; existing in idea or thought only; a mental conception regarded as a type or norm; a purely hypothetical construction.

FOUR SPACE—Often referred to as the fourth dimension (vide Chapter V); a space in which four coördinates (four lines drawn perpendicular to each other) are necessary and sufficient to establish the position of a point, as, a hypercube.

GAUSS, CHARLES FREDERICK, born at Brunswick, April 30, 1777. His father, being a bricklayer, had intended that he should follow the same occupation. So, in 1784, Charles was sent to the Bütner Public School in Brunswick, in order that he might be taught the ordinary elements of education. But during his attendance at this school, his unusual intelligence and aptitude attracted the attention and friendship of Professor BARTELS who later became the Professor of Mathematics at Dorpat. In 1792, through the kindly representations of Professor BARTELS to the Duke of Brunswick, young

Gauss was sent to the Collegium Carolinum. This greatly displeased his father as he saw in this move the frustration of his plans for Charles. In 1794, however, GAUSS entered the University of Göttingen still undecided whether he should make mathematics or philology his life work. While residing at Göttingen, he made his celebrated discoveries in analysis and these turned his attention definitely to the field of mathematics.

He completed his studies at Göttingen and returned to Brunswick in 1798, residing at Helmstadt where he had access to the Library in the preparation of his *Disquisitiones Arithmeticae* which was published in 1801. He received his doctorate degree (Ph.D.) on July 16, 1799. His next notable work was the invention of a method by which he calculated the elements of the orbit of the planet *Ceres* which had been discovered by PIAZZI, January 1, 1800, and who had left no record of his calculations by which other astronomers could locate the planet. GAUSS also calculated an ephemeris of *Ceres'* motion by means of which DE ZACH rediscovered the planet December 31, 1800.

His *Theoria Motus Corporum Coelestium in Conicis Sectionibus Solem Ambientium,* in which the author gives a "complete system of formulæ and processes for computing the movements of a heavenly body revolving in a conic section" is an outgrowth of his early researches and brought him lasting fame.

Through the influence of his friend OLBERS, he was appointed, July 9, 1807, first director of the new Göttingen Observatory, and Professor of Astronomy in the University, a position which he held until the end of his life. He died February 23, 1855. (Vide *Astronomical Society Notices,* Vol. 16, p. 80, 1856; also *Nature,* Vol. XV, pp. 533-537, 1877.)

GEOMETRISM—Of geometrical quality; a notion derived from PLATO'S declaration: "God geometrizes." It was his belief that the creative acts of the deity are executed in accordance with geometric design and laws; that in the totality of such acts there necessarily inheres a latent geometric quality.

KANT closely adhered to this notion in his discussions of space as an aspect of divine intelligence. He believed that the intellect merely rediscovers this latent geometrism when it turns to the study of materiality, and this belief is shared by BERGSON, the foremost metaphysician of the present time.

HYPERSPACE (hyper, above, beyond, transcending—space)—That species of space constructed by the intellect for convenience of measurement; an idealized construction; a purely arbitrary, conventional mathematical determination; the fourth dimension; any space that requires more than three coördinates to fix a point position in it, as, a five space, an *n*-space.

INTUITOGRAPH—The means by which the omnipsychic consciousness transmits intuitional impressions to the egopsychic or intellectual consciousness. An intuitogram is a direct cognition, an intuition; a primary truth projected into the egopsychic consciousness by the Thinker. It is recognized that, under the necessities of the present schematism of things, it is exceedingly difficult to propagate an intuition, especially with the same degree of ease as concepts are propagable; yet, this is believed to be a condition which will be overcome as the evolution of the higher faculties proceeds.

INVOLUTION—Process of enfolding, involving; antithesis of evolution; philosophically, the doctrine of involution maintains that, during the process of kosmic pyknosis (space-genesis), *all* that is to be expressed, developed and perfected as a result of the evolutionary movement was first involved, enfolded or deposited as latent archetypal tendencies and radicles in the original world-plasm; that, as the involutionary movement proceeded through the various phases of space-genesis, these became more and more phenomenal until at last they terminated in the elaboration of a manifest universe: each stage, accordingly, of the involutionary procedure became the basic substructure of a plane of specialized substance or ma-

teriality and consciousness. Thus it appears that evolution really begins where involution ends (vide Fig. 18), and the two opposing processes constitute the dualism of life as generating element. This notion has been symbolized in the *Lingam yoni* of Hellenistic philosophies, also in *Yang* and *Yin* of Chinese philosophy, which represent the original pair of opposites.

KATHEKOS—A purely arbitrary term devised for the express purpose of providing a convenient symbol to convey the idea embodied in the triglyph, *Chaos-Theos-Kosmos,* and is composed of the first three letters in each one of the terms of the triglyph; hence, symbolizes the triunity and interaction involved in the resolution of chaos into an orderly kosmos by the will of the Creative Logos. Thus, "kathekos" embodies a quadruplicate notion, namely, chaos, Creative Logos, manifested kosmos, and the creational activity of the Logos in the transmutation of disorder into order. The justification for this term, therefore, resides in its convenience, brevity and comprehensiveness.

By referring to figure 17, it will be seen that Kathekos divides into two kinds—involutionary, or that which pertains to involution, and evolutionary, or that that pertains to evolution. It thus comprises the beginning and the end of the world age or cycle and pertains to non-manifestation. The *raison d'être* of this differentiation is embodied in the notion that, on the involutionary arc of the cycle, the chaogenic period represents a phase of the world age when space-genesis is in an archetypal state wherein are involved all possibilities that are to become manifest in the kosmos, and on the evolutionary arc, the kathekotic period which is parallel to the chaogenic and represents a phase of the world age when the kosmos has reached ultimate perfection, embodying the perfected results of the possibilities which inhered in the chaogenic period or in involutionary kathekos. Thus, kathekos is dual in nature, on the one hand representing kosmic potency, and on the other, kosmic perfec-

tion of these potencies. It is Alpha, as related to involution, and Omega, as related to evolution.

KATHEKOSITY—A derivative, signifying creative activity and all that it implies; the state of consciousness or cognition corresponding thereto.

KLEIN, FELIX (1849—), born at Düsseldorf; studied at Bonn, and when only seventeen years of age was made assistant to the noted PLÜCKER in the Physical Institute. He took his doctorate degree in 1868, then went to Berlin, and later to Göttingen where he assisted in editing PLÜCKER'S works. He entered the Göttingen faculty in 1871; became Professor of Mathematics at Erlangen in 1872; and subsequently held professorships at Munich, 1875; Leipzig, 1880, and Göttingen, 1886. No one else in Germany has exerted so great influence upon American mathematics as he.

KOSMOS—*See Cosmos.*

LA GRANGE, JOSEPH LOUIS, born at Turin, January 25, 1736; died at Paris, April 10, 1813; regarded as the greatest mathematician since the time of Newton. It may be interesting to note that LA GRANGE remarked that mechanics is really a branch of pure mathematics analogous to a geometry of four dimensions, namely, time, and the three coördinates of the point in space. (Vide *Ball's Account of the History of Mathematics.*)

LIE, SOPHUS, a noted mathematician, referred to as the "great comparative anatomist of geometric theories, creator of the doctrines of Contact Transformations and Infinite Continuous Groups, and revolutionizer of the Theory of Differential Equations."

LOGOS—The supreme deity of the phenomenal universe; Creator; Fohat; a planetary god; the deity of a solar system.

MANVANTARA (Skt.)—A world age; the periods of involution and evolution combined; the stage during which the universe is in manifestation; a Day of Brahma.

MATHESIS (Gr. mathein, to learn)—Erudition; profound learning; the realm of metaphysical conceptions; the field of higher mathematics; the sphere of conceivability; the theoretical.

MENTOGRAPH—A cognitive factor consisting of a complete perception fused or in coalescence with a memory-image. Pure memory, of itself, is without utility as an aid to cognition; but, when nourished or supplemented by the substance of perception it becomes the basis of intellectual consciousness.

METAGEOMETRY (Gr. Meta, beyond, transcending —geometry)—Commonly, any kind of geometry that differs from the Euclidean, as the non-Euclidean; a geometry based upon the assumption that the angular sum of a triangle is greater or less than two right angles; the highest form of geometry; a system of idealized mathematical constructions. Sometimes called "pangeometry"; designated by GAUSS as "Astral Geometry"; the geometry of hyperspace. It consists of results arrived at by geometers in seeking a proof of the parallel-postulate.

META-SELF—The higher self in man; the universal self; the one self of which all individual selves are but fragments or parts. In man, it is coördinate with the omnipsyche (q.v.) and as such is the medium of kosmic consciousness.

MORPHOGENY (Gr. Morphe, form, vehicle, body—geny, evolution)—The evolution of forms, the production of individual bodies or vehicles for life, including organs and

faculties. Morphogenic—a derivative; pertaining to morphogeny; a kosmic process (vide figs. 17 and 18).

N-DIMENSIONALITY—Quality of conceptual space by virtue of which it may be regarded as possessing an indefinite number of dimensions.

NEAR-TRUTH—Any statement or view which is based upon partial knowledge; predicates concerning a class or genus derived from limited acquaintance with particulars of the class or genus; statements based upon logical determinations inhering in idealized constructions and applied to concrete or objective conditions; an abstraction viewed as a reality; the application of the qualities of abstractions to realities.

NEUROGRAM—Psychologically, a movement received by the afferent nerves in the form of a stimulation and transmitted through the brain and efferent nerves as either a reflex or voluntary action; a nerve impulse; a perception; a primary unit of intellectual consciousness; cf. *Intuitogram.*

NEWCOMB, SIMON (1835-1909), born at Wallace, Nova Scotia; educated in his father's school and came to the United States in 1853. Began, in 1854, teaching in Maryland; was appointed computer on *Nautical Almanac* at Cambridge in 1857; was graduated at Lawrence Scientific School in 1858; appointed Professor of Mathematics in the U. S. Navy in 1861. He supervised the construction of the 26-inch equatorial telescope at the Naval Observatory, and was secretary of the Transit of Venus Commission; was a member of nearly all of the Imperial and Royal societies of Europe and of the various societies in the United States, receiving the Copley Medal in 1874; the Huygens, 1878; the Royal Society, 1890, and the Bruce Medal in 1898; held the presidency of the following learned societies, viz: American Association for the Advancement of Science, 1877; Society for Psychical Re-

search, 1885-1886; American Mathematical Society, 1897-1898; the Astronomical and Astrophysical Society of America from its foundation in 1899. He rendered notable service in popularizing the doctrine of hyperspace.

NORM—An authoritative standard; model or type; standard of reference. The choice of a norm for spatial determinations cannot abide in any premise except that which naturally, and not artificially and conventionally, conforms to what is actually perceived; if so, there should be justification for challenging the wisdom and utility of the present schematism of things. There is an inherent conformity of space with intellect and intellect with space, and because of this natural complementarity of part with part and whole with whole, space cannot be otherwise than the intellectuality normally conceives it to be, provided, of course, that the cognitive movement is free and untrammeled by arbitrary hindrances. Consciousness, therefore, is the norm or standard of reference for all questions arising out of a consideration of spatiality.

OMNIPSYCHE—A term used to denote the Thinker's cognitive apparatus; the universal soul manifesting in individuals; the consciousness of the Thinker in virtue of which he is at-one with the universal consciousness; the medium of kosmic consciousness; the source of the intuition. cf. *Egopsyche*. The divinity in man (which is taken for granted), or his highest self can in no way be said justly to take its rise from sense-experience or from any bodily process. If divine, then eternal, and therefore, persistent. Broadly, the doctrine of evolution recognizes the passage of life from form to form, adding a little to each successive form and inevitably pushing each to a higher degree of perfection. Now, what is it that passes from form to form? Is it undifferentiated life or is it a specialized form of life? From every evidence, it would be judged that the life that ensouls an individual form is a specialized principle, i.e., limited to the execution of a given purpose.

If life as a specialized principle, limited to the execution of a given purpose in each form, passes on, it must preserve, at least, the sublimated results obtained during its residence in each individual form. It would thus become a sort of reservoir containing all these transmuted results. The omnipsyche, within the meaning of the text, is precisely this specialized life principle.

PARALLEL-POSTULATE—Variously referred to as the XIth, XIIth and XIIIth axiom of the *Elements of Euclid;* stated by MANNING, in *Non-Euclidean Geometry,* p. 91, in the following form: "If two lines are cut by a third, and the sum of the interior angles on the same side of the cutting line is less than two right angles, the lines will meet on that side when sufficiently produced." This celebrated postulate has proven to be the most fruitful ever devised; for it embodies in itself the possibility of three geometries based respectively upon the following assumptions, namely: I. That there exists a triangle, the sum of whose angles is congruent to a straight angle, the Euclidean; II. That there exists a triangle the sum of whose angles is less than a straight angle, the LOBACHEVSKIAN; III. That there exists a triangle the sum of whose angles is greater than a straight angle, the CAYLEY-KLEIN. Speaking of the content of the last two named, EDWARD MOFFAT WEYER,[1] says: "Hypothetical realms, wherein the dimensions of space are assumed to be greater in number than three, yield strange geometries, which are only card castles, products of a sort of intellectual play in the construction of which the laws of logic supply the rules of the game. The character of each is determined by whatsoever assumption its builder lays down at the start."

PASSAGE OF SPACE—A phrase connoting the movement of space from chaos to perfect order, a process believed to be infinite. The genesis of space necessarily implies an

[1] Vide *Popular Science Monthly,* vol. 78, p. 554, 1911.

elaboration, a procedure, by which the metamorphosis from disorder to kosmic order is made, and this movement is referred to as the "passage of space," a phenomenon thought to be measurable by means of a suitable instrumentality.

PERISOPHISM—See *Near-Truth.*

PSEUDOSPHERE—A surface of constant negative curvature; basis of BELTRAMI'S metageometrical calculations; surface resembling a champagne glass or common spool. The assumption that space is pseudospherical has given rise to the notion of space-curvature and various other conceptions.

PSYCHOGENY (Gr. Psyche, Soul—geny)—History of the evolution of the soul or the development of the sensomechanism in organisms. ERNST HAECKEL has traced the psychogeny of man through twenty-two different stages from the moneron to the anthropoid apes, and man.

PRALAYA (Skt.)—Kosmic quietude; the period during which the universe is not in manifestation; gestatory period; kosmic inactivity; opposed to manvantara (q.v.); figuratively, the kosmic womb; world egg.

PYKNON (Gr. pyknon, hard)—The principle of kosmic condensation; the primary basis of space-genesis; the initiation of the process by virtue of which chaos is elaborated into kosmic order. PYKNOSIS—The process of spatial engenderment. There are seven of these processes, each indicating a phase of duration, namely: MONOPYKNOSIS, the primary phase; DUOPYKNOSIS, secondary; TRIPYKNOSIS, tertiary. These three pertain to the plane of non-manifestation, the pralayic or gestatory duration-phase. The results arrived at during these duration-phases are concentrated in the Quartopyknotic which corresponds to the causal plane of manifestation or pure kosmic spirituality. QUINTOPYKNOSIS, a process con-

cerned in the genesis of mentality; SEXTOPYKNOSIS, kosmic sensibility; SEPTOPYKNOSIS, kosmic materiality. These seven phases of duration constitute the scope of space genesis or kosmogenesis, and incidentally depose the substructure of kosmic materiality, sensibility, intellectuality and spirituality, as well as the higher trinity of kosmic modes. The ramifications of these principles are innumerable and omnipresent. (See Chapter VII.)

QUARTODIM—A hypothetical being assumed to have a consciousness adapted to hyperspace or the fourth dimension, and whose scope of action is encompassed within a space which requires four coördinates, as the four-space.

REALITY (Realism)—Life; the harmony existing among the parts to maintain their equilibrium in the whole; the principle of integrity subsisting among parts; kosmic vitality.

RIEMANN, GEORGE FREDERICH BERNHARD, was born September 17, 1826, in the village of Breselenz, near Dannenburg, in Hanover. Until he was eight years of age his father was his sole tutor, but even at this age he exhibited great powers of arithmetical calculation. In the Spring of 1840 young RIEMANN was sent to the Hanover Lyceum where he remained for two years, leaving in 1842 for the Gymnasium at Luneburg. Here, under the direction of Professor SCHMALFUSS, he learned very rapidly, and is said to have required only one week thoroughly to familiarize himself with LEGENDRE'S *Theory of Numbers.*

On April 12, 1846 (Easter), he entered the University of Göttingen as a student of Theology in accordance with his father's wishes. His passion for mathematics, however, was so aroused by the lectures of GAUSS that he begged his father to be allowed to devote himself entirely to the studies of his choice. For two years he studied under JACOBI at Berlin. He then returned to Göttingen, and was graduated, his thesis being

a dissertation on the foundations of a general theory of functions of a variable complex magnitude. In 1854 he qualified as a teacher by giving a lecture on the "Hypothesis on which Geometry is Founded." In 1857 he became "Professor Extraordinarius," and in 1859 was elected Corresponding Member of the Academy of Sciences of Berlin and in 1860 a member of the Academy of Sciences of Göttingen.

After four years of failing health, during which he visited Messina, Palermo, Naples, Rome, Florence, Pisa and Milan, he died at Lago Maggiore, July 20, 1866, in full possession of his faculties and conscious of his approaching end.

SCHWEIKART, FERDINAND KARL (1780-1857), studied from 1796 to 1798 in Marburg, attending the mathematical lectures of J. K. F. HAUFF. In 1812 he became professor in Charkov, a position which he held for four years. In 1816 he became a tutor in the City of Marburg where he remained until 1820 when he transferred his labors to Königsberg. It was during his tutorship at Charkov, Marburg and Königsberg that he, entirely alone and without the slightest suggestion from any man, developed and taught a non-Euclidean geometry to the students under his care. For copy of his treatise on non-Euclidean geometry, see *Historical Sketch of the Hyperspace Movement,* Chapter II.

SCOPOGRAPHIC IMPRESSIONS—Sight perceptions fused with an associated memory-image, and forming the basis of action on external phenomena.

SENSOGRAPHIC IMPRESSIONS—Perceptions or impulses transmitted through the nerves of a sense-organ; any impression acting through the media of the senses.

SENSIBLE WORLD—The world of the senses; that which responds to the senses; the domain of perception; the phenomenal world; world of perceptual space.

SPACE-CURVATURE (see *Curvature of Space*).

SPACE-GENESIS—The process of spatial engenderment; the movement of life as engendering agent in bringing into manifestation the kosmos; the story of the appearance of the organized kosmos. The genesis of space can only be symbolized, as has been done in the text, for the limitations of human consciousness do not otherwise admit of the empirical establishment of the notion of its detailed procedure.

SPATIALITY—Space as a dynamic, creative movement; kosmic order, as opposed to disorder; the path of the engendering movement of life; the place of life. Spatiality, materiality, intellectuality and geometricity or the latent geometrism of the kosmos are thought of as being consubstantial and interdependent; but, of these, spatiality is regarded as the substance out of which the latter three are elaborated.

SUPERCONCEPTUAL—The purely intuitional; an act of cognition performed without the detailed work of conception derived from sense-data; conception of intuitions and their inter-relations; the Thinker's consciousness freed from intellectual characterization.

SUPERPERCEPTION—Perception of conceptual relations; a state of cognition wherein, instead of receiving percepts or images from the external world, then elaborating them into concepts, the Thinker apprehends composite images or concepts at first hand. It is a power which the liberated mind of the future will possess owing to the growing automatism of the intellect and the more facile expression of the intuitional consciousness.

TESSERACT (Gr. Tessera, four, cube, tessella)—A hypercube (see Chapter V.)

THINKER (Skt. Manu, thinker)—The real, spiritual man, as differentiated from his perceptive vehicles—mind, emotions and physical body; the omnipsychic intelligence who receives, classifies, interprets and preserves percepts; the manipulator of concepts; in fine, the higher, spiritual man.

The Thinker uses the various perceptive instrumentalities as so many tentacles or antennae by which he contacts the sensible world and makes the necessary adaptations to environment. He is the pure intelligence which is the source of all cognitive motivation; opposed to ego, because the egopsychic instrumentality is essentially an individualizing, separative agency; while the Thinker's omnipsychic intelligence is the basis of his unity with the universal intelligence. This conception of the Thinker implies that, as a spiritual intelligence, he is within and without the body, filling it as the ocean fills the sponge, encompassing, enveloping it and, at the same time, originating the totality of activities which manifest in and through the body. He is limited, therefore, in his manifestations in the sensible world only by the pliability of his vehicles.

TRANSFINITY—A state or condition that is incomprehensible to finite intelligence; that which transcends the finite, yet is not infinite; less than infinity and greater than finity. Space is referred to as being transfinite rather than infinite in extent. But space transfinite should be distinguished from space "finite though unbounded." For, there would seem to be little worthy of choice between a "finite, unbounded space" and an infinite one. The absence of boundary would naturally suggest an infinite extent. And although RIEMANN who is the author of the "unbounded" space arbitrarily determined that such a space should be a manifold possessing a measure of curvature which could be determined either by counting or actual measurement, he undoubtedly knew, nevertheless, that while each manifold might be an "unbounded" space the totality of such manifolds, infinite in number, must also be infinite in extent. It would seem to do violence to common sense, if not

to logical necessity, to view space both as "unbounded" and finite in extent, yet there would be no such difficulty in the recognition of space as being both transfinite and finite; because it is conceivable that the extent and character of space finite should transcend a finite intellectuality, and yet not be infinite.

TRIDIM—A being whose scope of consciousness is limited to a space of three dimensions, as ordinary human beings.
TRIDIMENSIONALITY—That quality possessed by perceptual space by virtue of which it is necessary and sufficient to have three coördinates, and only three, to establish the position of a point.

UNODIM—A hypothetical being assumed to have a consciousness limited to linear or one-space.

ZONES OF AFFINITY—Regions in the domain of intellectuality wherein minds, possessing a common differential, rate of vibration or quality, adhere to certain tenets from choice. Schools of philosophy, religions, and all those major divisions of intellectual effort which divide and subdivide intellectual allegiance are believed to take their rise in this property of intellectuality in virtue of which all minds having a similar coefficient gravitate towards a common agreement, especially where the movement is voluntary and untrammeled.

PART ONE

THE ESSENTIALS OF THE GEOMETRY OF HYPERSPACE AND THEIR SIGNIFICATIONS

CHAPTER I

The Prologue

On the Variability of Psychic Powers—The Discovery of the Fourth Dimension Marks a Distinct Stage in Psychogenesis—The Non-Methodical Character of Discoveries—The Three Periods of Psychogenetic Development—The Scope and Permissibility of Mathetic License—Kosmic Unitariness Underlying Diversity.

In presenting this volume to the public profound apologies are made to the professional mathematician for the temerity which is shown thereby. All technical discussion of the problems pertinent to the geometry of hyperspace, however, has been carefully avoided. The reader is, therefore, referred to the bibliography published at the end of this volume for matter relating to this aspect of the subject. The aim rather has been to outline briefly the progress of mathematical thought which has led up to the idea of the multiple dimensionality of space; to state the

cardinal principles of the Non-Euclidean geometry and to offer an interpretation of the metageometrical concept in the light of the evolutionary nature of human faculties and material characteristics and properties.

The onus of this treatise is, therefore, to distinguish between what is commonly known as sensible space and that other species of space known as geometric spaces. Also to show that the notion which has been styled *hyperspace* is nothing more nor less than an evidence of the faint, early outcroppings in the human mind of a faculty which, in the course of time, will become the normal possession of the entire human race. Thus the weight of all presentations will be to give currency to the belief, very strongly held, that humanity, now in its infancy, is yet to evolve faculties and capabilities, both mental and spiritual, to a degree hitherto viewed as inconceivable.

On this view it must appear that the faculty of thought including the powers of imagination and conceptualization are not psychological invariants, but, on the other hand, are true variants. They are, consequently, answerable to the principle of evolution just as all vital phenomena are. Some have thought that no matter what idea may come into the mind of the human race or at what time the idea may be born the mind always has been able to conceive it. That is, many believe that the nature of mind is such that no matter how complex an idea may be there has always been in the mind the power of conceiving it. But this view cannot be said to have the support of any trustworthy testimony. If so, then the mind must at once be recognized as fully matured and capable

during every epoch of human evolution, no less in the first than in the latest, which, of course, is absurd. It is undoubtedly more reasonable and correct to believe that the powers of conceptualization are matters of evolutionary concern. For instance, the assertion that the mind was incapable of conceiving, in the realm of theology, a non-anthropomorphic god, or, in the field of biology, the doctrine of evolution, or, in the domain of invention, the wireless telegraph, or, in mathematics, the concept of hyperspace before the actual time of these conceptions, cannot be successfully controverted.

In fact, it may be laid down as one of the first principles of psychogenesis that the mind rarely, if ever, conceives an idea until it has previously developed the power of conceptualizing it and giving it expression in the terms of prior experience. As in the growth of the body there are certain processes which require the full development of the organ of expression before they can be safely executed so in the phyletic development of faculties there are certain ideas, conceptions and scopes of mental vision which cannot be visualized or conceptualized until the basis for such mentation has been laid by the appearance of previously developed faculties of expression. And especially is this true of the intellect. Inasmuch as the entire content of the intellect is constituted of sense-derived knowledge, with the exception of intuitions which are not of intellectual origin though dependent upon the intellect for interpretation, there can be no doubt as to the necessity of there being first deposed in the intellect a sense-derived basis for intellection before it can become manifest. The Sen-

sationalists, led by LEIBNITZ, propounded as their fundamental premise this dictum: *"There is nothing in the intellect which has not first been in the senses except the intellect itself,"* and this has never been gainsaid by any school that could disprove it. The intuitionalist does not deny it: he merely claims that we are the recipients of another form of knowledge, the intuitional, which, instead of being derived from sense-experience, is projected into the intellectual consciousness from another source which we designate the Thinker. Thus, from the two forms of consciousness, come into the area of awareness truths that spring from entirely different sources. From the one source a steady stream of impressions flow constituting the substance of intellectual consciousness; from the other only a drop, every now and then, falls into the great inrushing mass so as to add a dim phosphorescence to an otherwise unilluminated pool. Obviously, when there is a lack of sensuous data from which a certain concept may be elaborated there can be no conception based upon them, and as the variety and quality of concepts are in exact proportion to the variety and quality of sense-experience there can be no demand for a particular species of notions such as might be elaborated out of the absent or non-existent perception. Hence, the power of conceiving springs forth from sense-experience. Sense-experience is essentially a mass of perceptions: these, creating a demand for additional adaptations, conspire, as if, to evoke the power or faculty to meet the demand, and consequently, an added conceptualization is made.

Progress in human thought is made in a manner

similar to that which prevails in the development of other natural processes, such as, the power of speech in the child. In the development of this faculty there are certain definite stages which appear in due sequence. The child is not gifted with the power of speech at once. It comes, by gradual and sometimes painful growth, into a full use of this faculty. Now, much the same principle holds true in the evolution of the mind in the human species. It is an established biologic principle that the ontogenetic processes manifested in the individual are but a recapitulation of the phylogenetic processes which are observable in the progress of the entire species. The view becomes even more cogent when note is taken of the fact that the foetus, during embryogenesis, passes successively through stages of growth which have been shown to be analogous, if not identical, with those stages through which the human species has developed, namely, the mineral, vegetal and animal.

Wherefore it may be said that the fourth dimensional concept marks a distinct stage in psychogenesis or evolution of mind. It required, as will be shown in Chapter II, nearly two thousand years for it to germinate, take root and come to full fruition. For it was not until the early years of the nineteenth century that mathematicians, taking inspiration from RIEMANN (1826-1866) fully recognized the concept as a metaphysical possibility, or even the idea was conceived at all. Serious doubt is entertained as to the possibility of its conception by any human mind before this date, that is, the time when it was actually born. Prior to that time, mathematical thought was taking upon itself that shape and tendence which would

eventually lead to the discovery of hyperspace; but it could not have reached the zenith of its upward strivings at one bound. That would have been unnatural.

Such is the constitution of the mind that although it is the quantity which bridges the chasm between the two stages of man's evolution when he merely thinks and when he really knows it is entirely under the domain of law and must observe the times and seasons, as it were, in the performance of its functions. The scope of psychogenesis is very broad, perhaps unlimited; but its various stages are very clearly defined notwithstanding the breadth of its scope of motility. And while the distance from *moneron* to man, or from feeling to thinking is vast, the gulf which separates man, the Thinker, from man, the knower, is vaster still. Who, therefore, can say what are the delights yet in store for the mind as it approaches, by slow paces, the goal whereat it will not need to struggle through the devious paths of perceiving, conceiving, analyzing, comparing, generalizing, inferring and judging; but will be able to know definitely, absolutely and instantaneously? That some such consummation as this shall crown the labors of mental evolution seems only natural and logical.

It may be thought by some that the character and content of revelational impressions constitute a variation from the requirements of the law above referred to, but a little thought will expose the fallacy of this view. The nature of a revealed message is such as to make it thoroughly amenable to the restrictions imposed by the evolutionary aspects of mind in general. That this is true becomes apparent upon an examination of the four cardinal characteristics of such

impressions. First, we have to consider the indefinite character of an apocalyptic ideograph which is due to its symbolic nature. This is a feature which relieves the impression of any pragmatic value whatsoever, especially for the period embracing its promulgation. Then, such cryptic messages may or may not be understood by the recipient in which latter case it is nonpropagable. Second, the necessity of previous experience in the mind of the recipient in order that he may be able to interpret to his own mind the psychic impingement. The basis which such experience affords must necessarily be present in order that there may be an adequate medium of mental qualities and powers in which the ideogram may be preserved. A third characteristic is that revelations quite invariably presuppose a contemplative attitude of mind which, in the very nature of the case, superinduces a state of preparedness in the mind for the proper entertainment of the concept involved. This fact proves quite conclusively that revelational impressions are not exceptions to the general rule. Lastly, a dissatisfaction with the conditions with which the symbolism deals or to which it pertains is also a prerequisite. This condition is really that which calls forth the cryptic annunciation, and yet, preceding it is a long series of causes which have produced both the conditions and the revolt which the revelator feels at their presence. In view of the foregoing, it would appear that objections based upon the alleged nonconformity of the revealed or inspired cannot be entertained as it must be manifest that it, too, falls within the scope of the laws of mental growth.

Discoveries, whether of philosophical or mechanical

nature, or whether of ethical or purely mathematical tendence, are never the results of a deliberate, methodical or purposive reflection. For instance, let us take LIE'S "transformation groups," mathematic contrivances used in the solution of certain theorems. Now, it ought to be obvious that these mathetic machinations were not discovered by SOPHUS LIE as a consequence of any methodic or purposeful intention on his part. That is, he did not set out deliberately to discover "transformation groups." For back of the "groups" lay the entire range of analytic investigations; the mathematical thought of more than a thousand years furnished the substructure upon which LIE built the conception of his "groups." Similarly, it may be said with equal assurance that no matter how great the intensity of thought, nor how purposeful, nor of how long duration the series of concentrated abstractions which led up to the invention of the printing press, the linotype or multiplex printing press of our day could not have been produced abruptly, nor by use of the mental dynamics of the human mind of remoter days. Its production had to follow the path outlaid by the laws of psychogenesis and await the development of those powers which alone could give it birth. The whole question resolves itself, therefore, into the idea of the complete subserviency of the mind, in all matters of special moment, to the laws aforementioned. The supersession of the law of its own life by the mind is well-nigh unthinkable, if not quite so.

If we now view the history of the mind as manifested in the human species, three great epochs which divide the scope of mental evolution into more or

less well-defined stages present themselves. These are: first, the *formative stage;* second, *the determinative stage;* third, the stage of *freedom,* or the *elaborative* stage.

In all of the early races of men, through every step which even preceded the *genus homo,* the generic mind was being formulated. It was being given shape, outline and direction. All of the first stage, the *formative,* was devoted to organization and direction. Those elementary sensations which constituted the basis of mind in the primitive man were accordingly strongly determinative of what the mind should be in these latter days. To this general result were contributed the effects of the activity of cells, nerves, bones, fibers, muscles and the blood.

The *formative* period naturally covered a very extensive area in the history of mind or psychogenetic development. It was followed closely, but almost insensibly, by the *determinative* period during which all the latent powers, capacities and faculties which were the direct products of the *formative* period were being utilized in meeting the demands of the law of necessity. The making of provisions against domestic want, against the attacks of external foes; the combating of diseases, physical inefficiency, the weather, wild beasts, the asperities of tribal enmities; as well as furthering the production of art, music, sculpture, the various branches of handiwork, literature, philosophies, religions and the effectuation of all those things which now appear as the result of the mental activity of the present-day man make up the essence and purpose of the determinative period.

Signs of the dawn of the *elaborative* stage, also

called the stage of *freedom,* have been manifest now for upwards of three centuries and it is, therefore, in its beginnings. It is not fully upon us. Not yet can we fully realize what it may mean, nor can we unerringly forecast its ultimate outcome; but we feel that it is even now here in all the glories of its matutinal freshness. And the mind is beginning to be free from the grinding necessities of the constructive period having already freed itself from the restrictive handicaps of the primeval formulation period. Already the upgrowing rejuvenescences so common at the beginning of a new period are commencing to show themselves in every department of human activity in the almost universal desire for greater freedom. And this is particularly noticeable in the many political upheavals which, from time to time, are coming to the surface as well as in the countless other aspects of the wide-spread renaissance. Perhaps the time may come, never quite fully, when there will be no longer any necessity to provide against the external exigencies of life; perhaps, the time will never be when the mind shall no more be bound by the law of self-preservation, not even when it has attained unto the immortality of absolute knowledge; yet, it is intuitively felt that it must come to pass that the mind shall be vastly freer than it is to-day. And with this new freedom must come liberation from the necessities of the elementary problems of mere physical existence.

The inference is, therefore, drawn that the fourth dimensional concept, and all that it connotes of hyperspace or spaces of *n*-dimensionality are some of the evidences that this stage of freedom is dawning. And the mind, joyous at the prospect of unbounded liberty

which these concepts offer, cannot restrain itself but has already begun to revel in the sunlit glories of a newer day. What the end shall be; what effect this new liberty will have on man's spiritual and economic life; and what it may mean in the upward strivings of the Thinker for that sublime perpetuity which is always the property of immediate knowledge no one can hope, at the present time, to fathom. It is, however, believed with KEYSER that "it is by the creation of hyperspaces that the rational spirit secures release from limitation"; for, as he says, "in them it lives ever joyously, sustained by an unfailing sense of infinite freedom."

The elevating influence of abstract thinking, such as excogitation upon problems dealing with entities inhabiting the domain of *mathesis* is, without doubt, incalculable in view of the fact that it is only through this kind of thought that the spirit is enabled to reach its highest possibilities. This is undoubtedly the philosophy of those religious and occult exercises known as "meditations," and this perhaps was the main idea in the mind of the Hebrew poet when he exclaimed: "Let the words of my mouth and the meditation of my heart be acceptable in thy sight, O Lord, my strength and my Redeemer." The principal, if not the only, value possessed by the "summitless hierarchies of hyperspaces" which the mathematician constructs in the world of pure thought is the enrichening and ennobling influence which they exert upon the mind. But admittedly this unbounded domain of mathetic territory which he explores and which he finds "peopled with ideas, ensembles, propositions, relations and implications in endless variety

and multiplicity" is quite real to him and subsists under a reign of law the penalties of which, while not as austere and unreasonable as some which we find in our tridimensional world, are nevertheless quite as palpable and as much to be feared. For the orthodoxy of mathematics is as cold and intolerant as ever the religious fanatic can be. But the reality and even the actuality which may be imputed to the domain of mathesis is of an entirely different quality from that which we experience in our world of triune dimensionality and it is a regrettable error of judgment to identify them. It ought, therefore, never be expected, nor is it logically reasonable to assume that the entities which inhabit the mathetic realm of the analyst should be submissive to the laws of sensible space; nor that the conditions which may be found therein can ever be made conformable to the conditions which exist in perceptual space.

It was PLATO's belief that ideas alone possessed reality and what we regard as actual and real is on account of its ephemerality and evanescence not real but illusionary. This view has been shared by a number of eminent thinkers who followed, with some ostentation, the lead established by PLATO. For a considerable period of time this school of thinkers had many adherents; but the principles at length fell into disrepute owing to the absurdities indulged in by some of the less careful followers. The realism, or for that matter, the actuality of ideas cannot be denied; yet it is a realism which is neither to be compared with the physical reality of sense-impressions nor its phenomena. The character and peculiarity of ideas are in a class apart from similar notions of perceptual

space content. It is as if we were considering the potentialities of the spirit world and the entities therein in connection with incarnate entities which in the very nature of the case is not allowable. Furthermore, it is unreasonable to suppose that the conditions on a higher plane than the physical can be made responsible to a similar set of conditions on the physical plane.

There are certain astronomers who base their speculations as to the habitability of other planets upon the absurd hypothesis that the conditions of life upon all planets must be the same as those on the earth, forgetting that the extent of the universe and the scope of motility of life itself are of such a nature as to admit of endless variations and adaptations. There is a realism of ideas and a realism of perceptual space. Yet this is no reason why the two should be identified. On the other hand, owing to the diversity in the universe, every consideration would naturally lead to the assumption that they are dissimilar. To invest ideas, notions, implications and inferences with a reality need not logically or otherwise affect the reality of a stone, a fig, or even of a sense-impression.

To a being on the spirit levels our grossest realities must appear as non-existent. They are neither palpable nor contactable in any manner within the ordinary range of physical possibilities. For us his gravest experiences can have no reality whatsoever; for no matter how real an experience may be to him it is altogether beyond our powers of perception, and therefore, to us non-existent also. It should, however, be stated that the state of our knowledge about a

given condition can in no way affect its existence. It merely establishes the fact that two or more realities may exist independent of one another and further that the gamut of realism in the universe is infinite and approaches a final state when its occlusion into absolute being follows as a logical sequence.

Recurring to the consideration of the reality of spirit-realms as compared with that of sensible space, it comes to view that our idealism, that is, the idealism which is a quality of conceptualization, may be regarded as identical with their realism, at least as being on the same plane as it. Stated differently, the things that are ideal to us and which constitute the data of our consciousness may be as real to them as the commonest object of sense-knowledge is to us. What, therefore, appears to us as the most ethereal and idealistic may have quite a realistic character for them.

Ultimately, however, and in the final deeps of analysis it will be found undoubtedly that both our realism and our idealism as well as similar qualities of the spirit world are in all essential considerations quite illusionary. All knowledge gained in a condition short of divinity itself is sadly relative. Even mathematical knowledge falls far short of the absolute, the fondest claims of the orthodox mathematician to the contrary notwithstanding. It has been said frequently that a mathematical fact is an absolute fact and that its verity, necessity and certainty cannot be questioned anywhere in the universe whether on Jupiter, Neptune, Fomalhaut, Canopus or Spica. But having so declared, the fact of the sheer relativity of our knowledge is not disturbed thereby nor controverted.

Happily, neither distance nor a lack of distance can in any way affect the quality of human knowledge, mathematical knowledge not excepted. That can only be affected by conditions which cause it to approach perfection and nothing but evolution can do that.

In the light of results obtained in analytic investigations the question of the flexibility of mathematical applications becomes evident and one instead of being convinced of the vaunted invariability of the laws obtaining in the world of mathesis is, on the other hand, made aware of the remarkable and seemingly unrestrained facility with which these laws may be made to apply to any conditions or set of assumptions within the range of the mind's powers of conception. Mathematicians have deified the *definition* and endowed it with omnific powers imputing unto it all the attributes of divinity—immutability, invariance, and sempiternity. In this they have erred grievously although, perhaps, necessarily. Mathetic conclusions are entirely conditional and depend for their certainty upon the imputed certitude of other propositions which in turn are dependent, in ever increasing and endlessly complex relations, upon previously assumed postulates. These facts make it exceedingly difficult to understand the attitude of mind which has obscured the utter mutability and consequent ultimate unreliability of the fine-spun theories of analytic machinations.

The apriority of all mathematical knowledge is open to serious questioning. And although there is no hesitancy in admitting the basic agreement of the most primary facts of mathematical knowledge with the essential character of the intellect the existence of well-defined limits for such congruence cannot be

gainsaid. The subjunctive quality of geometric and analytical propositions is made apparent by an examination of the possibilities falling within the scope of permissibility offered by mathetic license. For instance, privileged to proceed according to the analytic method it is allowable to reconstruct the sequence of values in our ordinary system of enumeration so as to admit of the specification of a new value for say, the entire series of odd numbers. This value might be assumed to be a plus-or-minus one, dependent upon its posture in the series. That is, all odd numbers in the series beginning with the digit 3, and continuing, 5, 7, 9, 11, 13, 15, 17, 19, . . . *n*, could be assumed to have only a place value which might be regarded as a constant-variable. The series of even numbers, 2, 4, 6, 8, 10, 12, 14, 16, . . . *n*, may be assumed to retain their present sequence values. Under this system the digit 1 would have an absolute value; all other odd numbers would have a constant-variable value; constant, because always no more nor less than 1 dependent upon their place in the operations and whether their values were to be applied by addition or subtraction to or from one of the values in the even number series; variable, because their values would be determinable by their application and algebraic use.

There would, of course, be utilitarian objection to a system of this kind; but under the conditions of a suppositionary hypothesis, it would be self-consistent throughout, and if given universal assent would suit our purposes equally as well as our present system. But the fact that this can be done under the mathematic method verily proves the violability of mathe-